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Intrinsic mass-richness relation of clusters in THE THREE HUNDRED hydrodynamic simulations

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Outline

Background

- Cluster of galaxies
- Mass richness (MR) relation
- Halo occupation distribution (HOD)

Method

- Model

Data

- The Three Hundred
- Cluster catalogue

Results

- MR relation using galaxy stellar mass
- MR relation using galaxy magnitude

Discussions

- comparison with other models
- 7-parameters relation
- comparison with previous work

Conclusions

1. Background

Background — Cluster of galaxies

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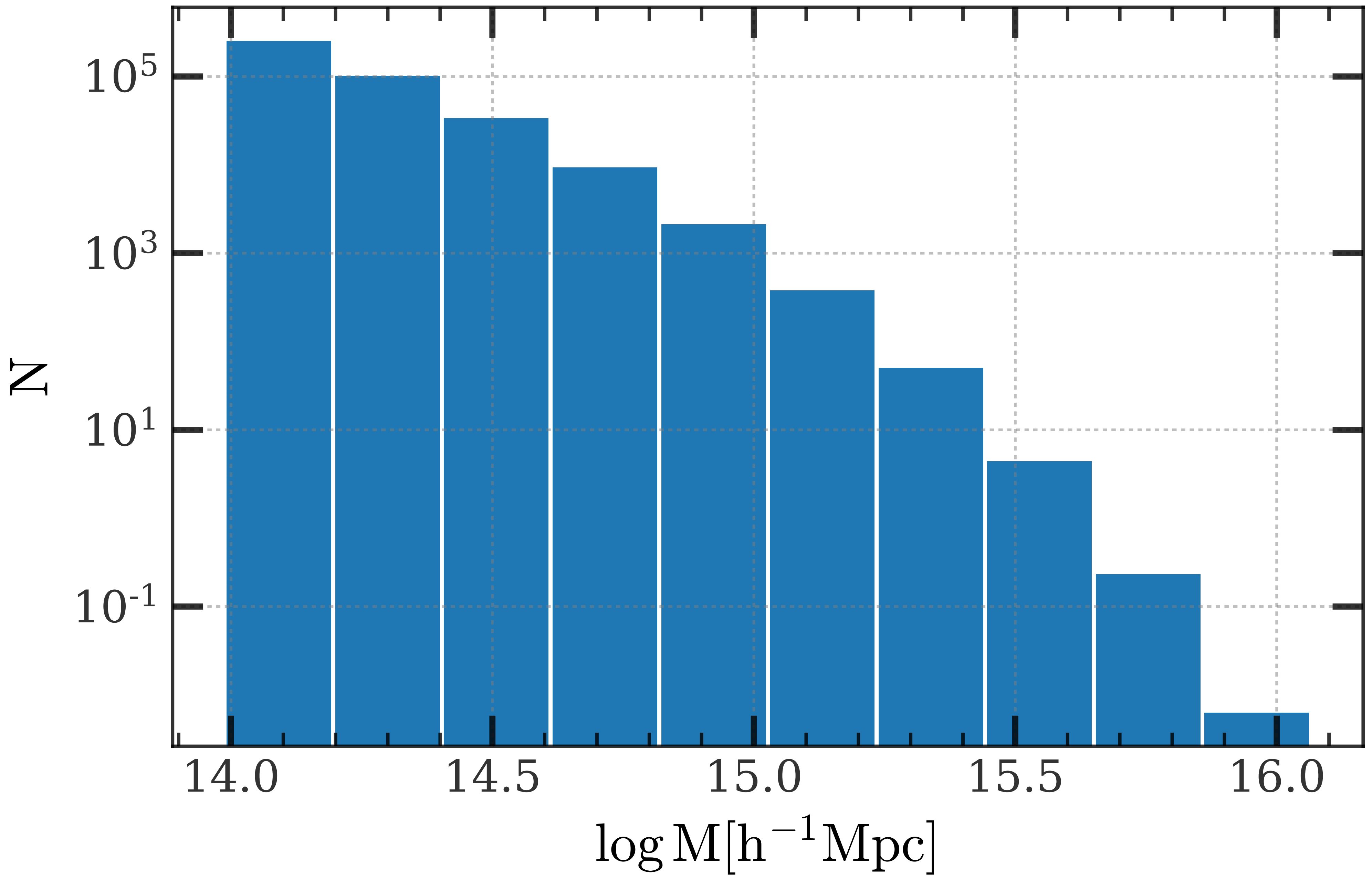
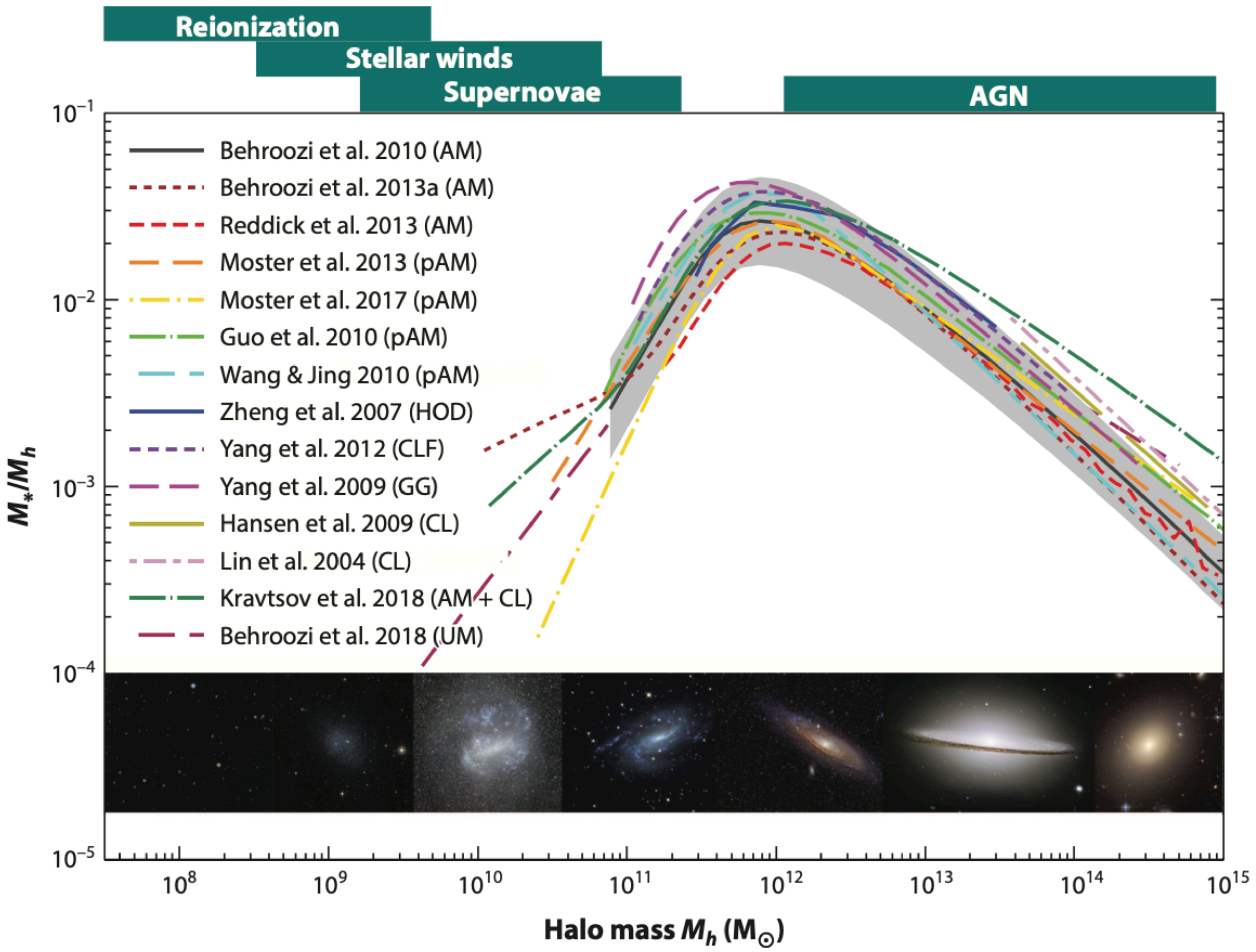
Cosmology

- Cluster abundance $N(M, z)$
- Cluster power spectrum
- Cluster stacked lensing



Galaxy formation and evolution

- Central galaxy
  - Brighter, redder, and more concentrated centrals reside in more massive clusters
- Satellite galaxy
  - Baryonic process: harassment, ram-pressure stripping, tidal stripping, dynamical friction, and strangulation



Background — Cluster(halo) mass

Individual cluster

1. Dynamics
   - Assumption: dynamical equilibrium
   - galaxy number density profile $\nu(r)$, galaxy velocity dispersion $\sigma(r)$

$$M(r) = -\frac{r\sigma^2(r)}{G}\left[\frac{d\ln\sigma^2(r)}{d\ln r} + \frac{d\ln\nu(r)}{d\ln r} + 2\beta\right]$$

2. X-ray
   - Assumption: hydrostatic equilibrium
   - Gas density profile $n(r)$, gas temperature profile $T(r)$

$$M(r) = -\frac{rkT(r)}{G\mu m_p}\left[\frac{d\ln n(r)}{d\ln r} + \frac{d\ln T(r)}{d\ln r}\right]$$

3. Lensing
   - No assumptions
   - Shear profile $\gamma(\theta)$

require high-quality or long-term spectral observations

direct observables as mass proxies
mass-observable relation

Background — Cluster(halo) mass

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Mass proxies

- X-ray:
  - Gas mass M_{gas}
  - Gas temperature, luminosity T_X, L_X
  - Integrated Y_X
- Millimeter:
  - Integrated SZ flux Y_{SZ}
- Optical:
  - Galaxy overdensity
  - Luminosity L
  - Richness λ

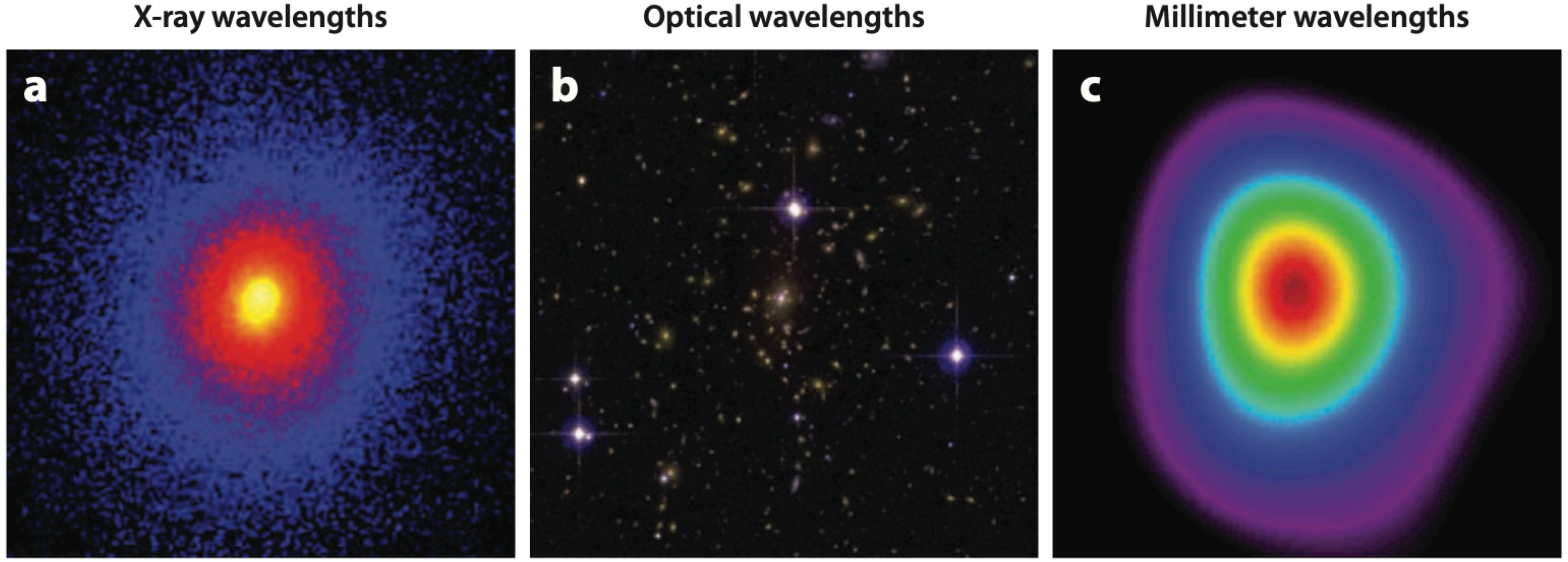
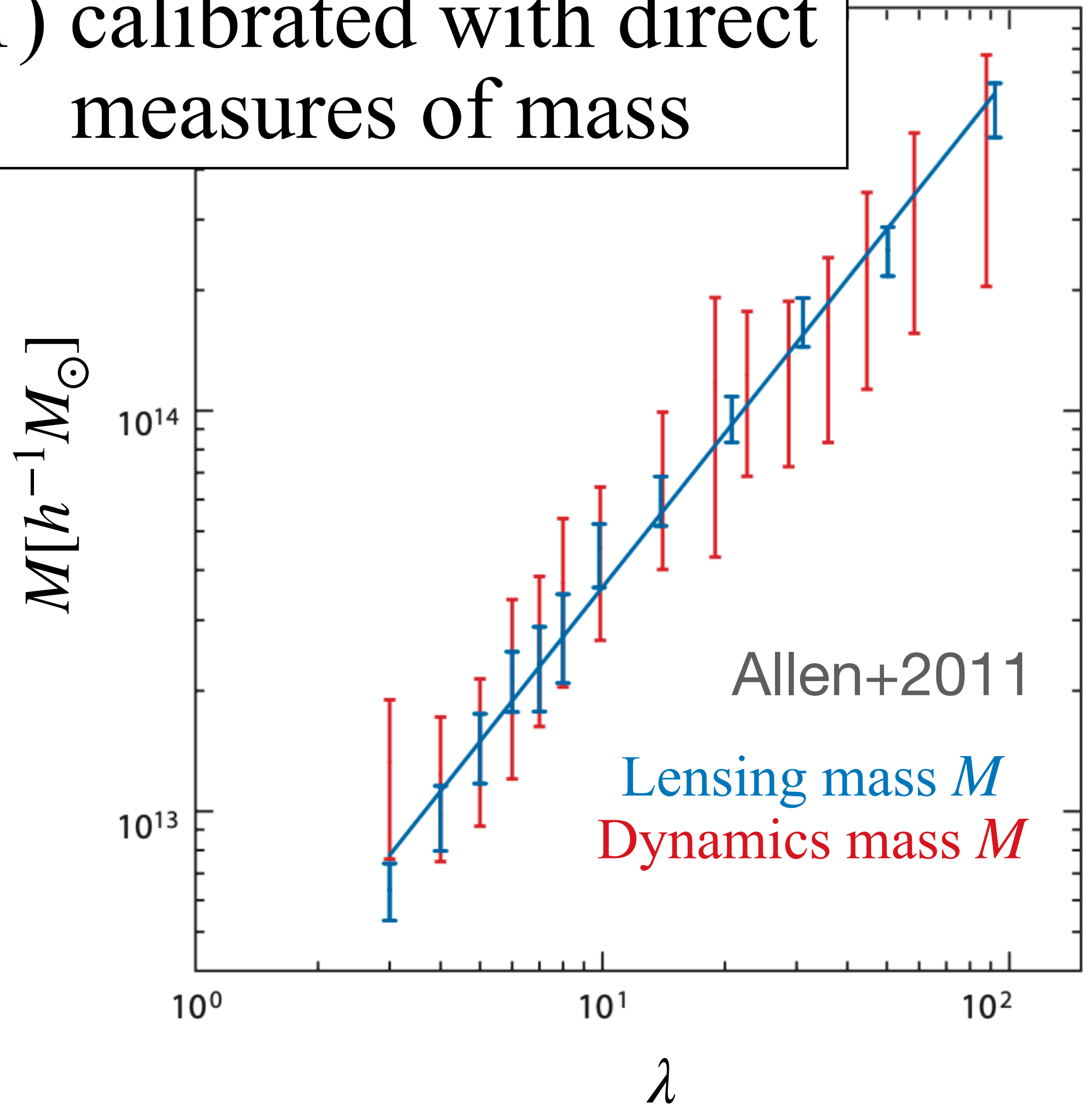
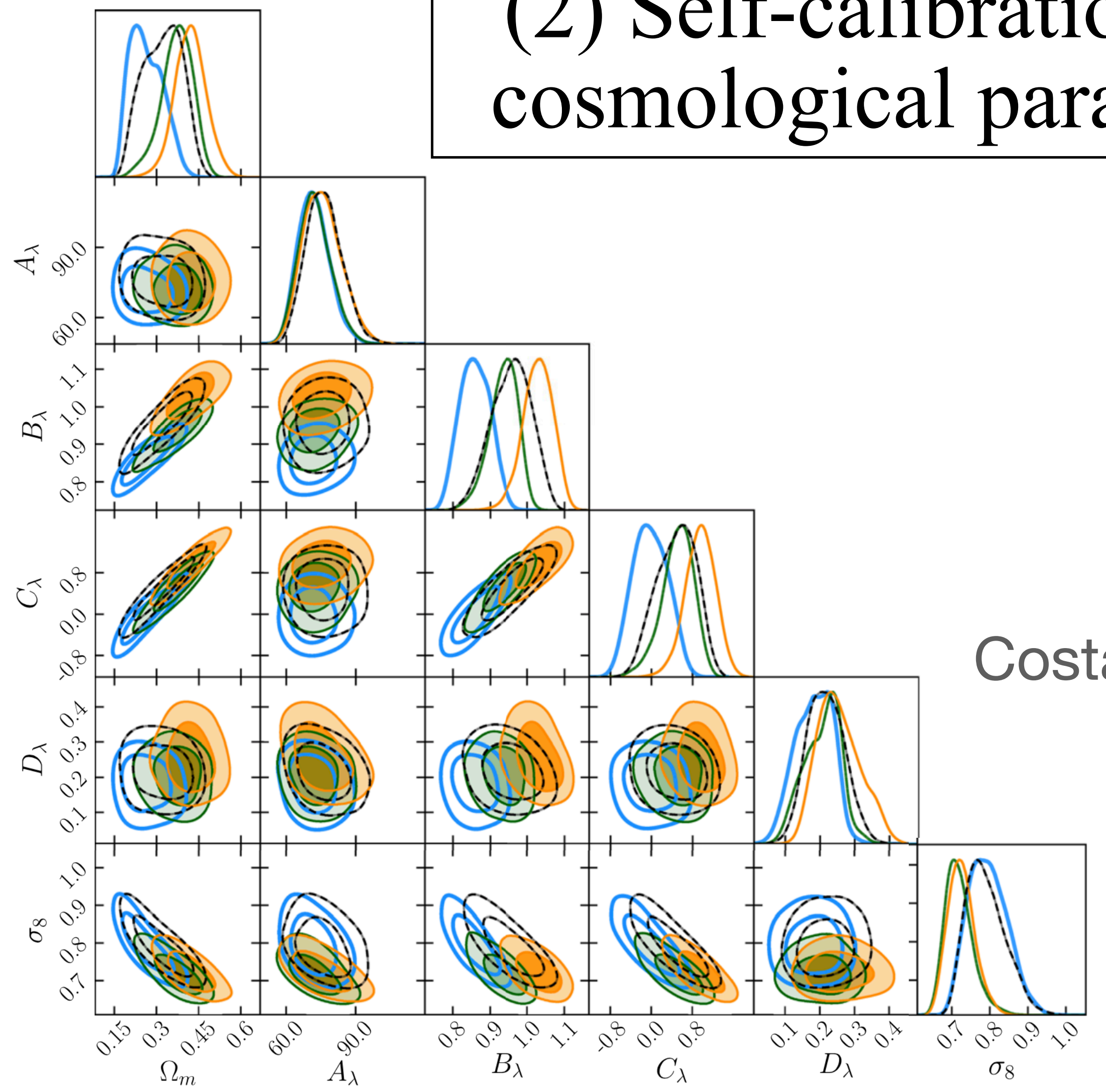


Fig. Abell 1835 cluster

(1) calibrated with direct measures of mass



(2) Self-calibration with cosmological parameters



Costanzi+2021

Mean relation $\lambda(M)$

- Power-law

$$\langle \ln \lambda \,|\, \ln M \rangle = A + B \ln \left(\frac{M}{M_{piv}} \right)$$

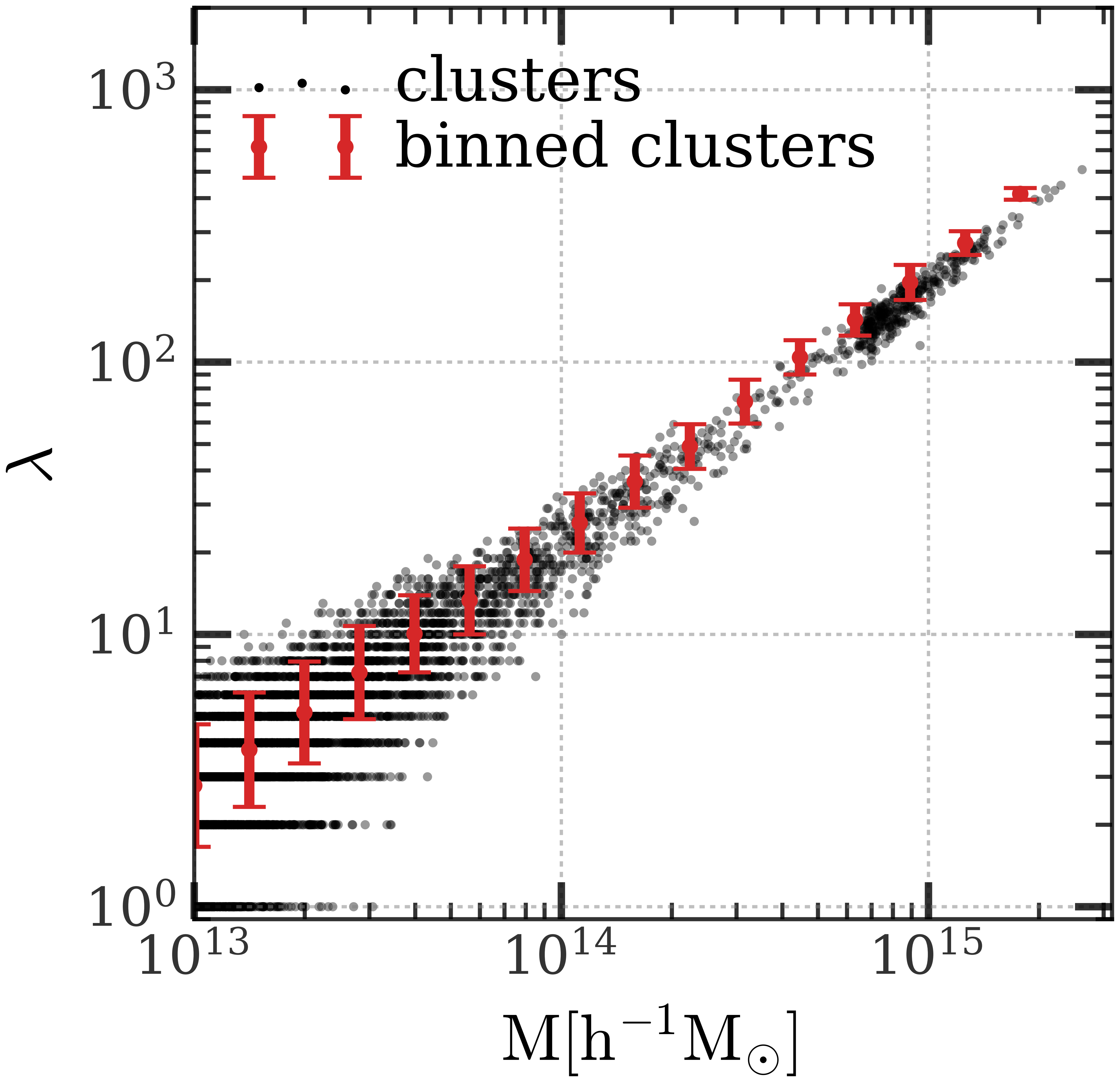
Richness PDF $P(\lambda \,|\, M)$

Probability Distribution Function

- Log-normal

$$P(\ln \lambda \,|\, \ln M) = \frac{1}{\sqrt{2\pi}\sigma_{\ln \lambda | \ln M}} \exp \left[-\frac{(\ln \lambda - \langle \ln \lambda \,|\, \ln M \rangle)^2}{2\sigma^2_{\ln \lambda | \ln M}} \right]$$

$$\sigma_{\ln \lambda} \sim M?$$



Background — MR relation

Mass Richness relation

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Scatter $\sigma_{\ln\lambda}$

1. Simple linear relation

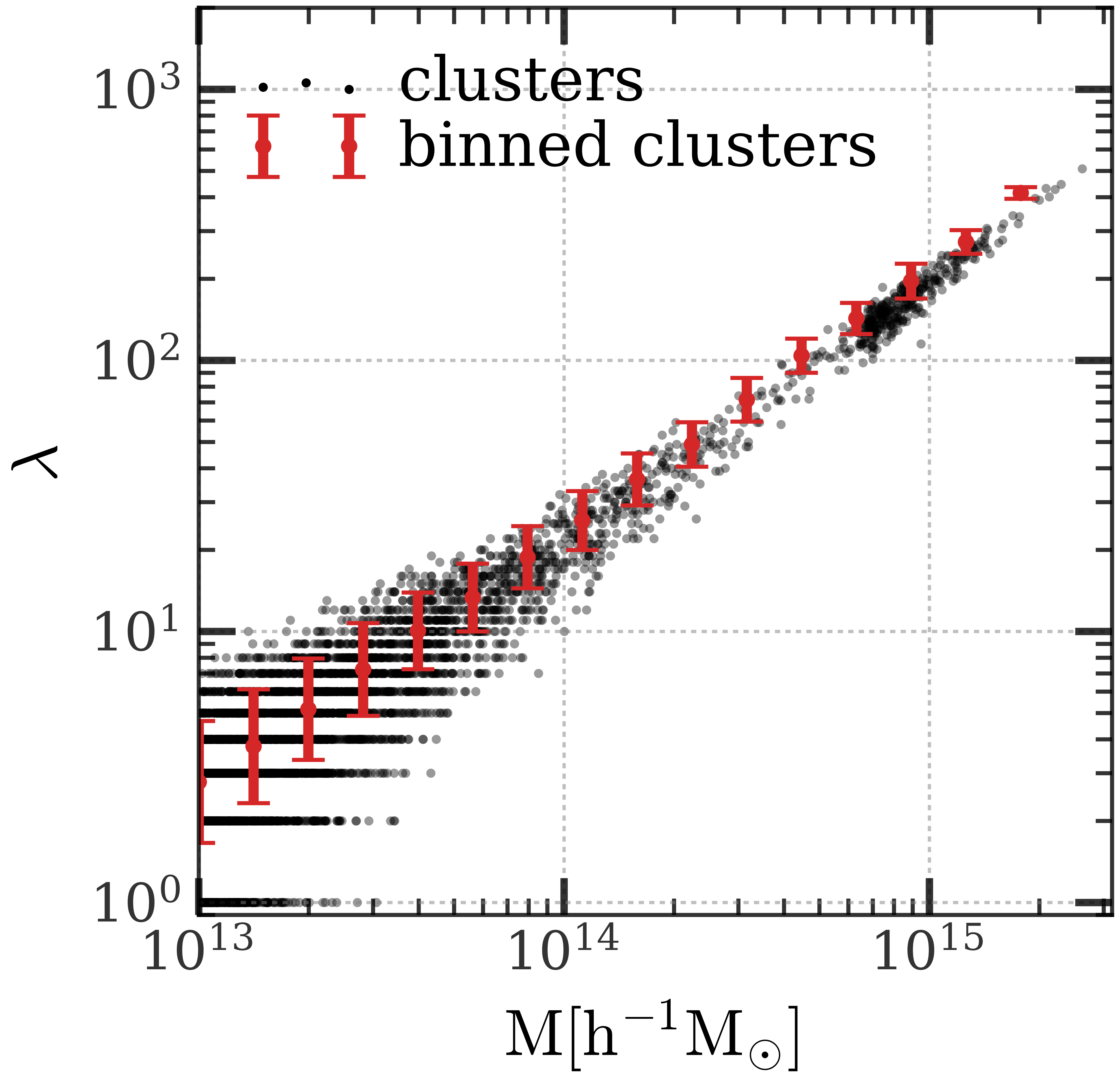
$$\sigma_{\ln\lambda} = \sigma_0 + q \ln\left(\frac{M}{M_p}\right)$$

Murata+2018(SDSS), Murata+2019(HSC)

2. Intrinsic scatter + Poisson term

$$\sigma^2_{\ln\lambda} = \sigma^2_{IG} + \frac{e^{\langle\ln\lambda\rangle} - 1}{e^{2\langle\ln\lambda\rangle}}$$

Capasso+2019(ROSITA), Bleem+2020(SPT), Costanzi+2021(DES+SPT), To+2021(DES)



Background — HOD

Halo Occupation Distribution

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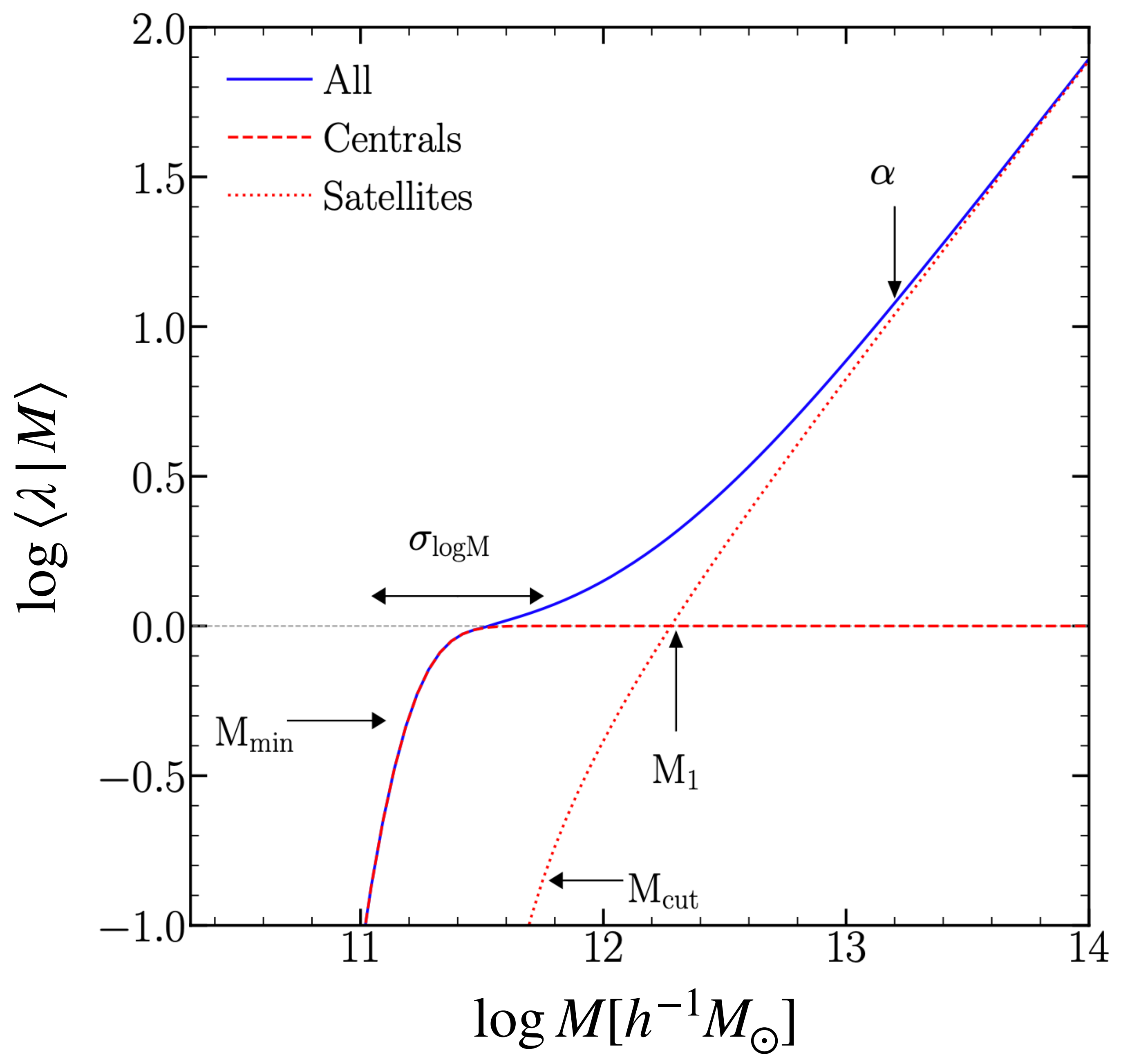
$\langle \lambda | M \rangle$ - 5 parameters

- Central - 2 parameters $\{M_{min}, \sigma_{\log M}\}$

$$\langle \lambda^{cen} | M \rangle = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{\log M - \log M_{min}}{\sigma_{\log M}}\right)\right]$$

- Satellite - 3 parameters $\{M_{cut}, M_1^*, \alpha\}$

$$\langle \lambda^{sat} | M \rangle = \left(\frac{M - M_{cut}}{M_1^*}\right)^{\alpha}$$



Contreras +2017

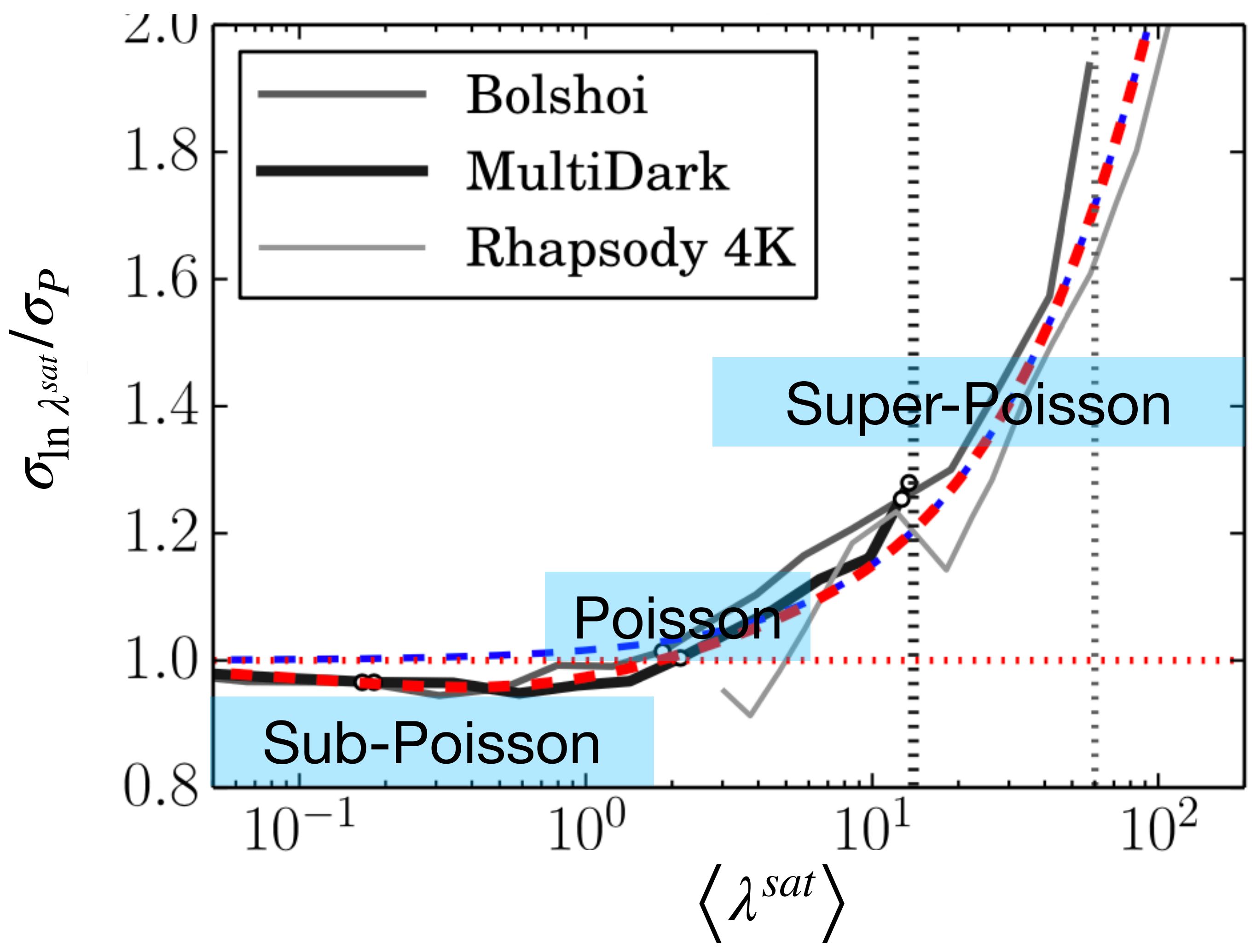
$P(\lambda^{sat} | M)$ - Sub-Poisson, Poisson, Super-Poisson

- Super-Poisson at large mass

σ_P Poisson scatter: statistics of halo merger histories,

σ_I Super, intrinsic scatter: halo-to-halo scatter

arises from variance in the large-scale environments of the host haloes



Mao+2015

2. Method

Method — Model

Mean relation $\lambda(M)$

- Power-law

$$\langle \ln \lambda \,|\, \ln M \rangle = A + B \ln \left(M/M_{piv} \right)$$

Richness PDF $P(\lambda \,|\, M)$

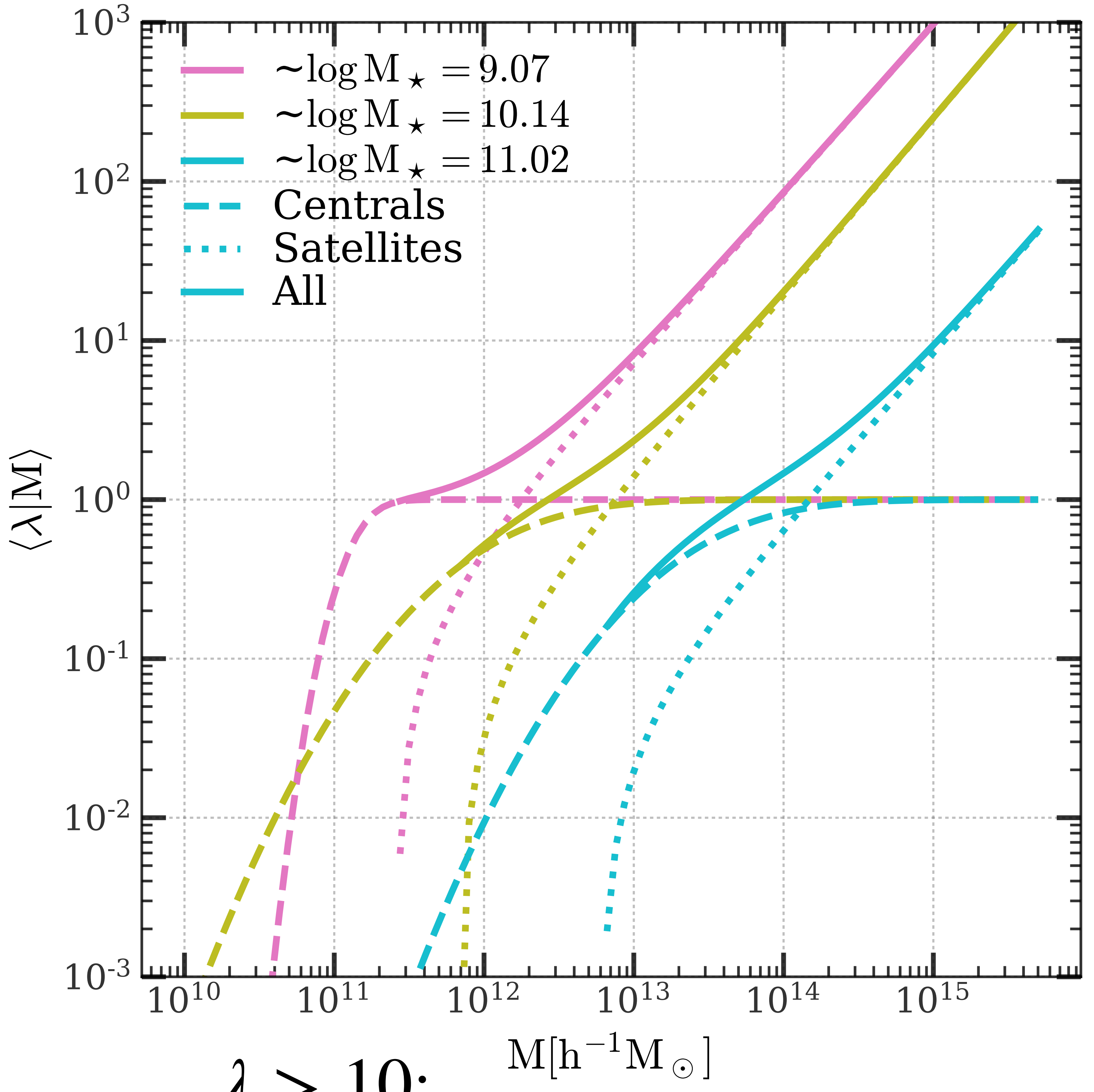
Probability Distribution Function

☒ Log-normal

$$P(\ln \lambda \,|\, \ln M) = \frac{1}{\sqrt{2\pi}\sigma_{\ln \lambda|\ln M}} \exp\left[-\frac{(\ln \lambda - \langle \ln \lambda \,|\, \ln M \rangle)^2}{2\sigma^2_{\ln \lambda|\ln M}} \right]$$

☑ $P(\lambda \,|\, M)$ a convolution of a Poisson σ_P distribution with a Gaussian σ_I distribution.

- no analytic closed form



Contreras +2017

$\lambda > 10$:

- power-law
- Super-Poisson

Model σ_I as Gaussian

Method — Model

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Mean relation $\lambda(M)$

- Power-law

$$\langle \ln\lambda \,|\, \ln M\rangle = A + B\ln\left(M/M_{piv}\right)$$

Richness PDF $P(\lambda\,|\,M)$

Probability Distribution Function

☒ Log-normal

$$P(\ln\lambda\,|\,\ln M) = \frac{1}{\sqrt{2\pi}\sigma_{\ln\lambda|\ln M}}\exp\left[-\frac{(\ln\lambda - \langle\ln\lambda\,|\,\ln M\rangle)^2}{2\sigma^2_{\ln\lambda|\ln M}}\right]$$

☑ $P(\lambda\,|\,M)$ a convolution of a Poisson σ_P distribution with a Gaussian σ_I distribution.

- no analytic closed form

- Model σ_I as Gaussian

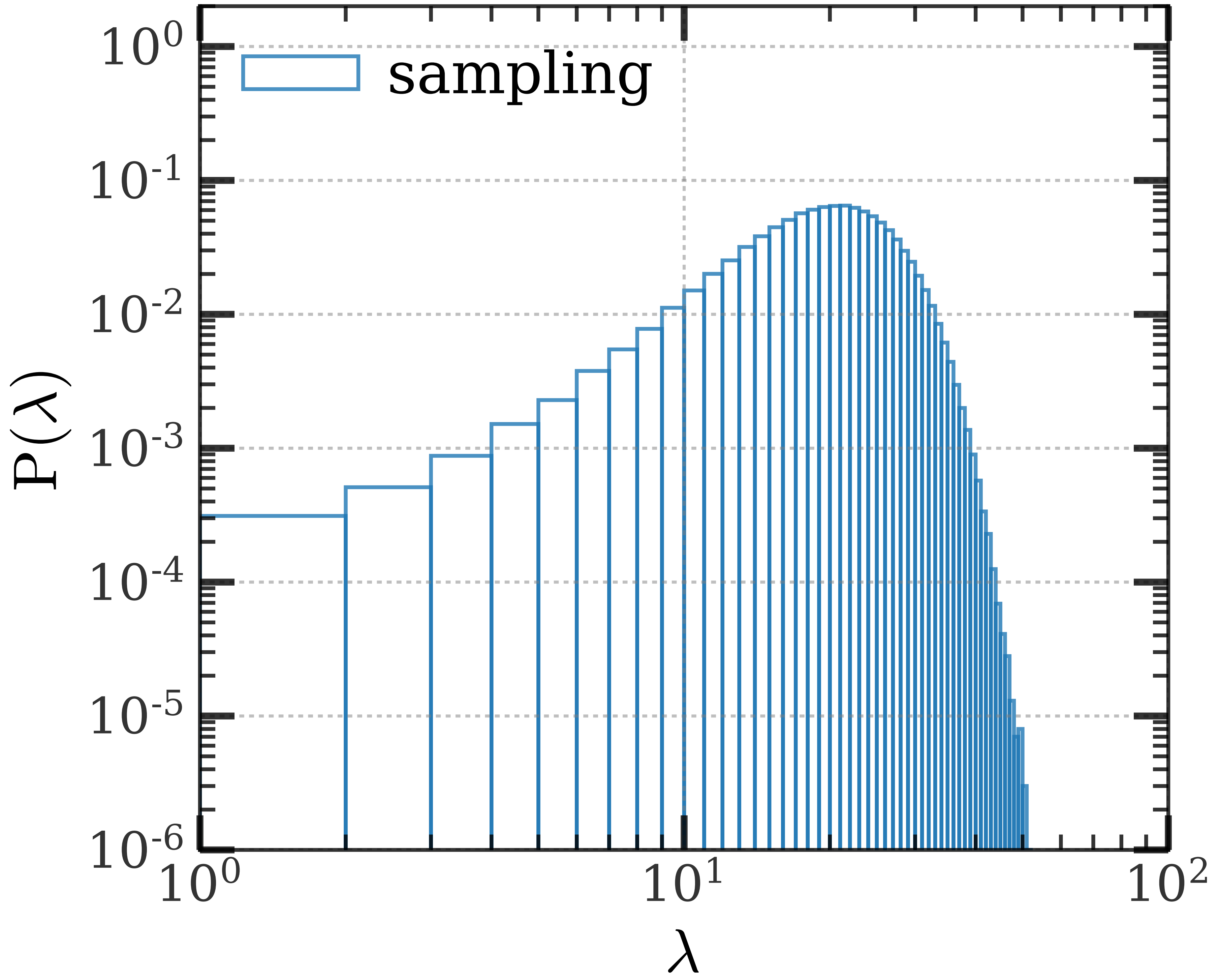
$$\lambda = 1 + \Delta^P + \Delta^G$$

$$\Delta^P \sim Poission(\langle\lambda^{sat}\rangle)$$

$$\Delta^G \sim Gaussian(0,\sigma_I)$$

Sample 10^6 λ

* $\langle\lambda^{sat}\rangle = 20, \sigma_I = 0.2 \quad => \quad P(\lambda)$



Method — Model

Mean relation $\lambda(M)$

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$$\langle \ln \lambda \,|\, \ln M \rangle = A + B \ln \left(M/M_{piv} \right)$$

Richness PDF $P(\lambda \,|\, M)$

Probability Distribution Function

☒ Log-normal

$$P(\ln \lambda \,|\, \ln M) = \frac{1}{\sqrt{2\pi}\sigma_{\ln \lambda | \ln M}} \exp \left[-\frac{(\ln \lambda - \langle \ln \lambda \,|\, \ln M \rangle)^2}{2\sigma^2_{\ln \lambda | \ln M}} \right]$$

☑ $P(\lambda \,|\, M)$ a convolution of a Poisson σ_P distribution with a Gaussian σ_I distribution.

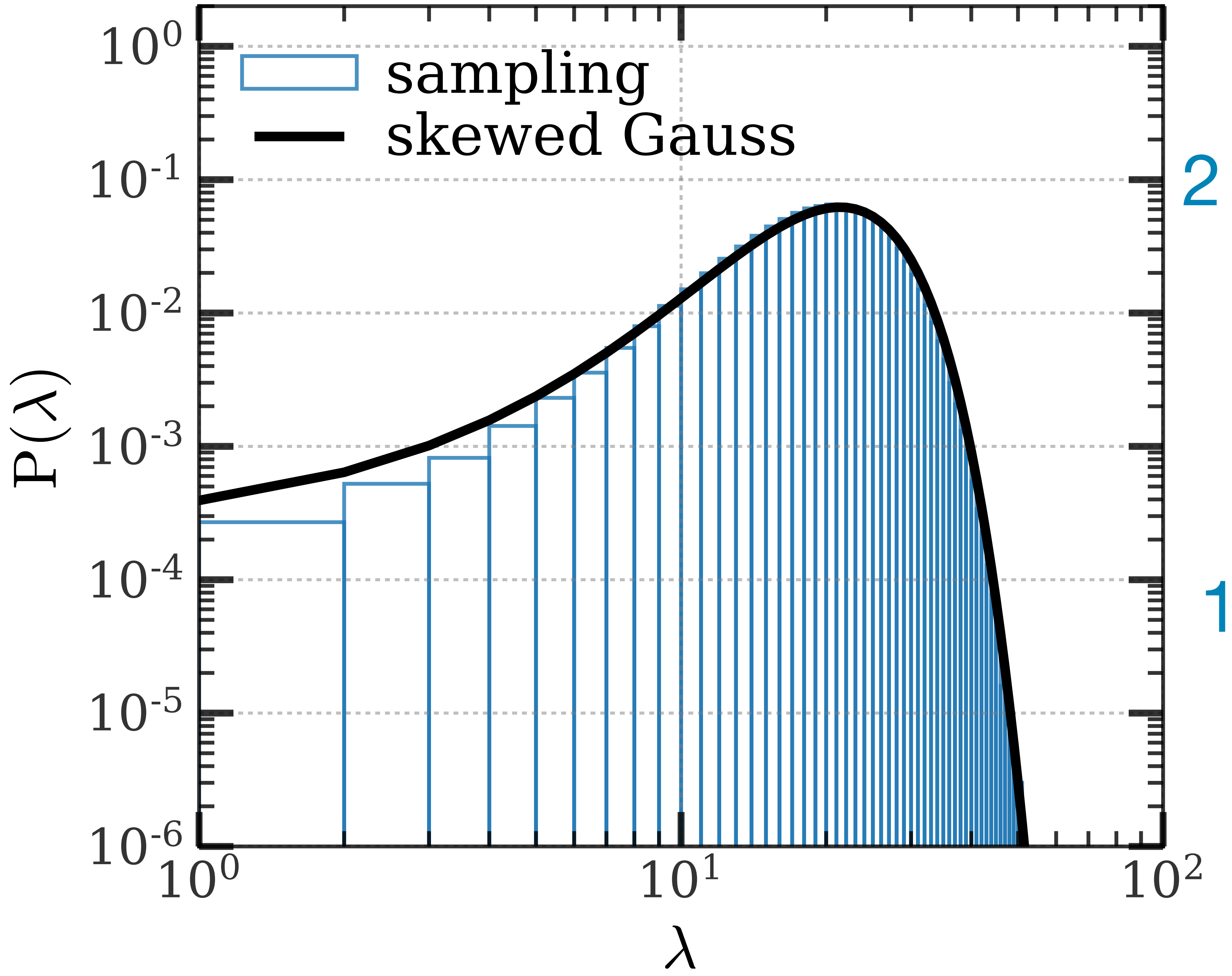
- no analytic closed form
- use a skewed Gaussian function to fit it.

- use a skewed Gaussian function to fit it

$$P(\lambda \,|\, M) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(\lambda - \langle \lambda^{sat} | M \rangle\right)^2}{2\sigma^2}} \operatorname{erfc} \left[-\alpha \frac{\lambda - \langle \lambda^{sat} \,|\, M \rangle}{\sqrt{2\sigma^2}} \right]$$

Sample 10^6 Fit

* $\langle \lambda^{sat} \rangle = 20, \sigma_I = 0.2$ => $P(\lambda)$ => $\sigma = 6.55, \alpha = 0.27$



2 parameters $\{\sigma, \alpha\}$

↓

1 parameter σ_I

Method — Model

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Mean relation $\lambda(M)$

- Power-law

$$\langle \ln \lambda \,|\, \ln M \rangle = A + B \ln \left(M / M_{piv} \right)$$

3 parameters:

$$\{A, B, \sigma_I\}$$

Richness PDF $P(\lambda \,|\, M)$

Probability Distribution Function

☒ Log-normal

$$P(\ln \lambda \,|\, \ln M) = \frac{1}{\sqrt{2\pi}\sigma_{\ln \lambda | \ln M}} \exp\left[-\frac{(\ln \lambda - \langle \ln \lambda \,|\, \ln M \rangle)^2}{2\sigma^2_{\ln \lambda | \ln M}} \right]$$

1. Simple linear relation: $\sigma_{\ln \lambda} = \sigma_0 + q \ln \left(M / M_p \right)$
2. Intrinsic scatter + Poisson term : $\sigma^2_{\ln \lambda} = \sigma^2_{IG} + \frac{e^{\langle \ln \lambda \rangle} - 1}{e^{2\langle \ln \lambda \rangle}}$

☑ Skewed Gaussian

$$P\left(\lambda \,|\, M \right) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(\lambda - \langle \lambda^{sat} | M \rangle \right)^2}{2\sigma^2}} \operatorname{erfc}\left[-\alpha \frac{\lambda - \langle \lambda^{sat} \,|\, M \rangle}{\sqrt{2\sigma^2}} \right]$$

3. Intrinsic scatter: σ_I

3. Data

Data — The Three Hundred

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- The most massive ($M > 8 \times 10^{14}\ h^{-1}M_{\odot}$) 324 clusters are selected from the MultiDark simulation(MDPL2)

  MDPL2: DM-only, 1 $h^{-1}Gpc$, 3840^3 DM, $m_{DM} = 1.5 \times 10^9\ h^{-1}M_{\odot}$

- 324 zoomed-in initial conditions are generated by cutting a spherical region with a radius of 15 $h^{-1}Mpc$

  $m_{DM} + m_{gas} = 1.5 \times 10^9\ h^{-1}M_{\odot}$ $\Omega_M = 0.307, \Omega_b = 0.048$

  $m_{DM} = 12.7 \times 10^8\ h^{-1}M_{\odot}$, $m_{gas} = 2.36 \times 10^8\ h^{-1}M_{\odot}$

- hydrodynamical simulations with baryonic models:

  GADGET-X: calibrated based on gas properties

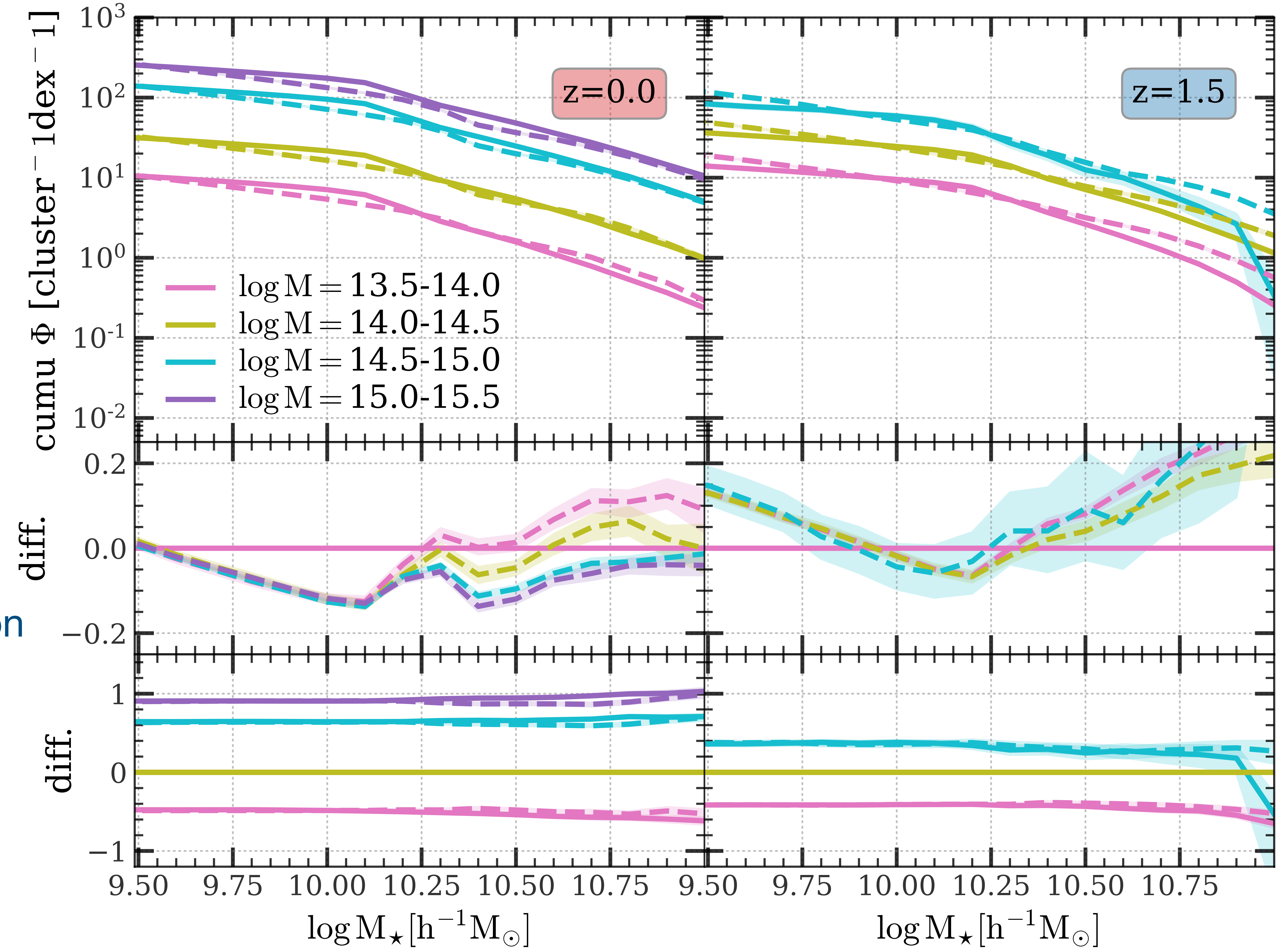
  GIZMO-SIMBA: calibrated based on the stellar properties

Data — Catalogue

- 324 regions
- Redshift: $z = 0, 0.5, 1, 1.5$

- Halo finder: AHF (SO)
  - Cluster mass: $M \equiv M_{200c}$
- Galaxy finder: Caesar (6DFOF)
  - Galaxy stellar mass: $M_\star$
  - Galaxy absolute magnitude $\mathscr{M}$ in different bands:
    - CSST i-band: $\mathscr{M}_i$
    - CSST z-band: $\mathscr{M}_z$ Chinese Space Station Telescope
    - Euclid h-band: $\mathscr{M}_h$ a European Space Agency mission

Cumulative satellite stellar mass function $\Phi(M_\star)$



- Cumulative satellite stellar mass function $\Phi(M_\star)$
  - GADGET-X and GIZMO-SIMBA are more consistent at small $M_\star$

Data — Catalogue

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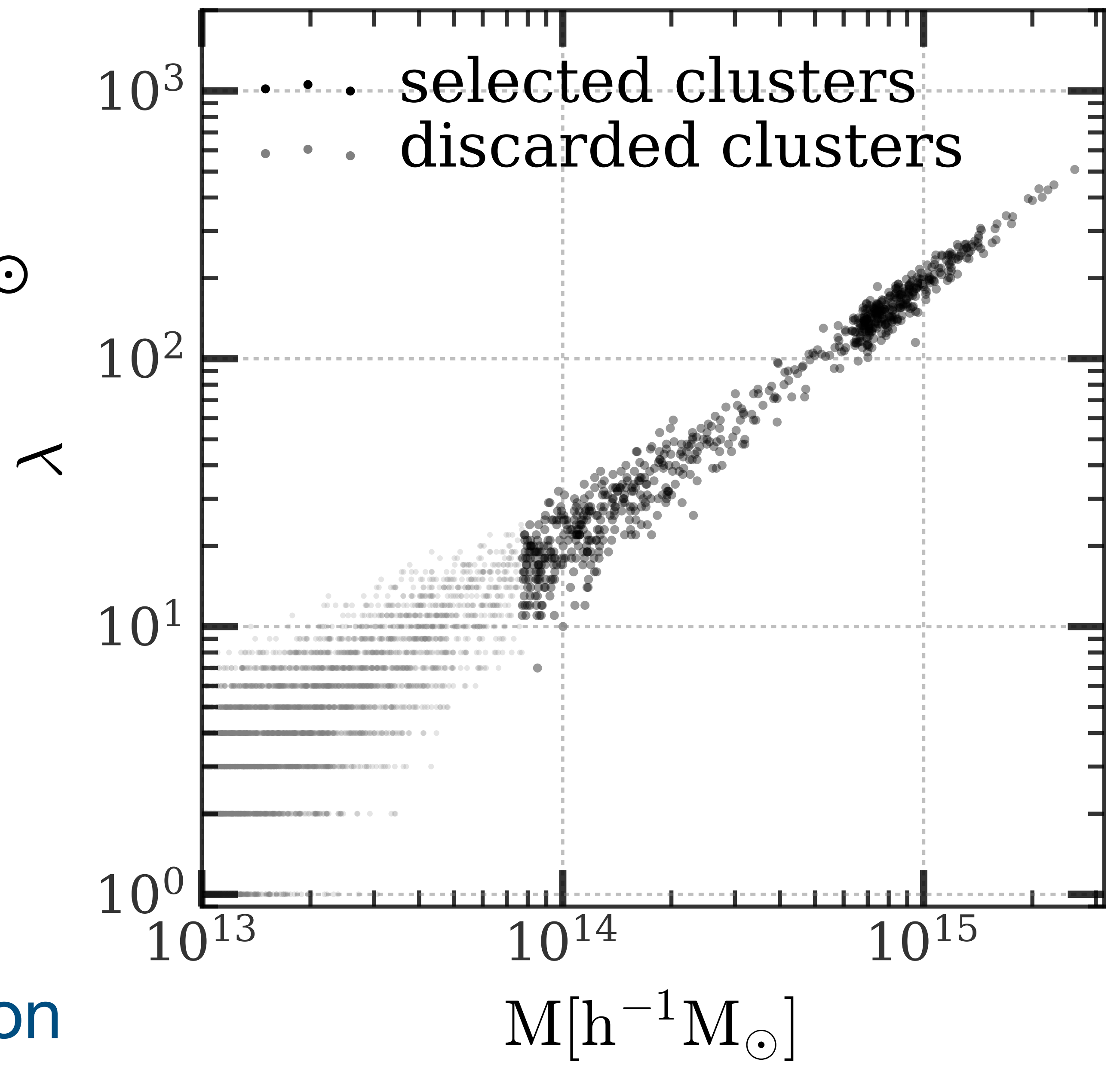
- 324 regions
- Redshift: $z = 0, 0.5, 1, 1.5$

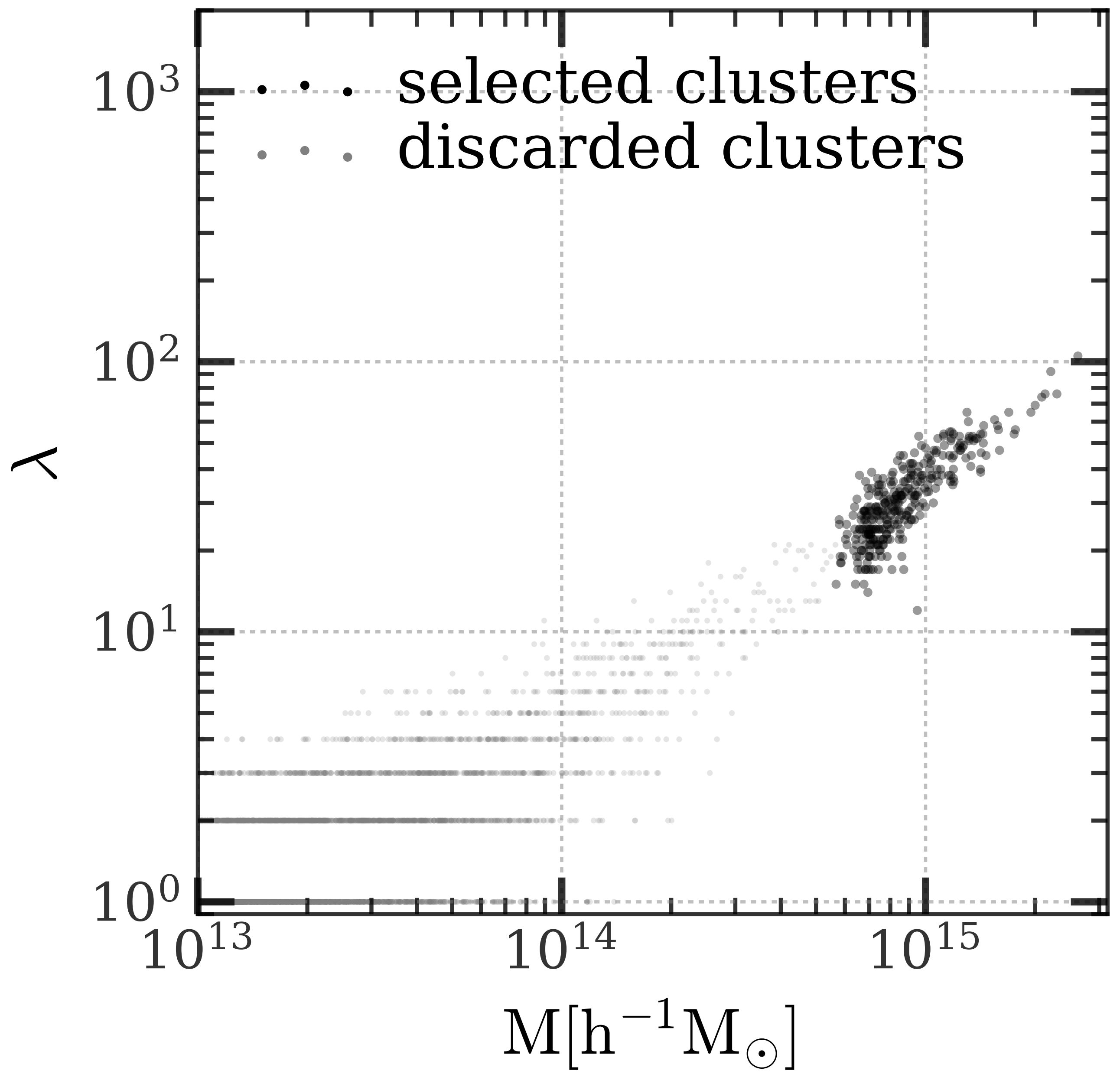
- Halo finder: AHF (SO) $\sim \lambda > 10$
  - Cluster mass: $M \equiv M_{200c} \gtrsim 5 \times 10^{13} \sim 6 \times 10^{14}\ h^{-1}M_\odot$
- Galaxy finder: Caesar (6DFOF)
  - Galaxy stellar mass: $M_\star \geq 10^{9.5}\ h^{-1}M_\odot$ Resolution
  - Galaxy absolute magnitude $\mathscr{M}$ in different bands:
    - CSST i-band: $\mathscr{M}_i$
    - CSST z-band: $\mathscr{M}_z$ Chinese Space Station Telescope
    - Euclid h-band: $\mathscr{M}_h$ a European Space Agency mission

- Richness Definition: λ

  the count of member galaxies selected by:
  - $M_\star$
  - $\mathscr{M}$



z=0
GADGET-X
$M_\star \geq 10^{9.5}\ h^{-1}M_\odot$
Selected clusters: 752



z=0
GADGET-X
$M_\star \geq 10^{10.5}\ h^{-1}M_\odot$
Selected clusters: 333

4. Results

Results — MR relation using $M_\star$

Power-law: $\langle \ln \lambda \mid \ln M \rangle = A + B \ln \left(M/M_{piv} \right)$

Skewed Gaussian PDF: σ_I

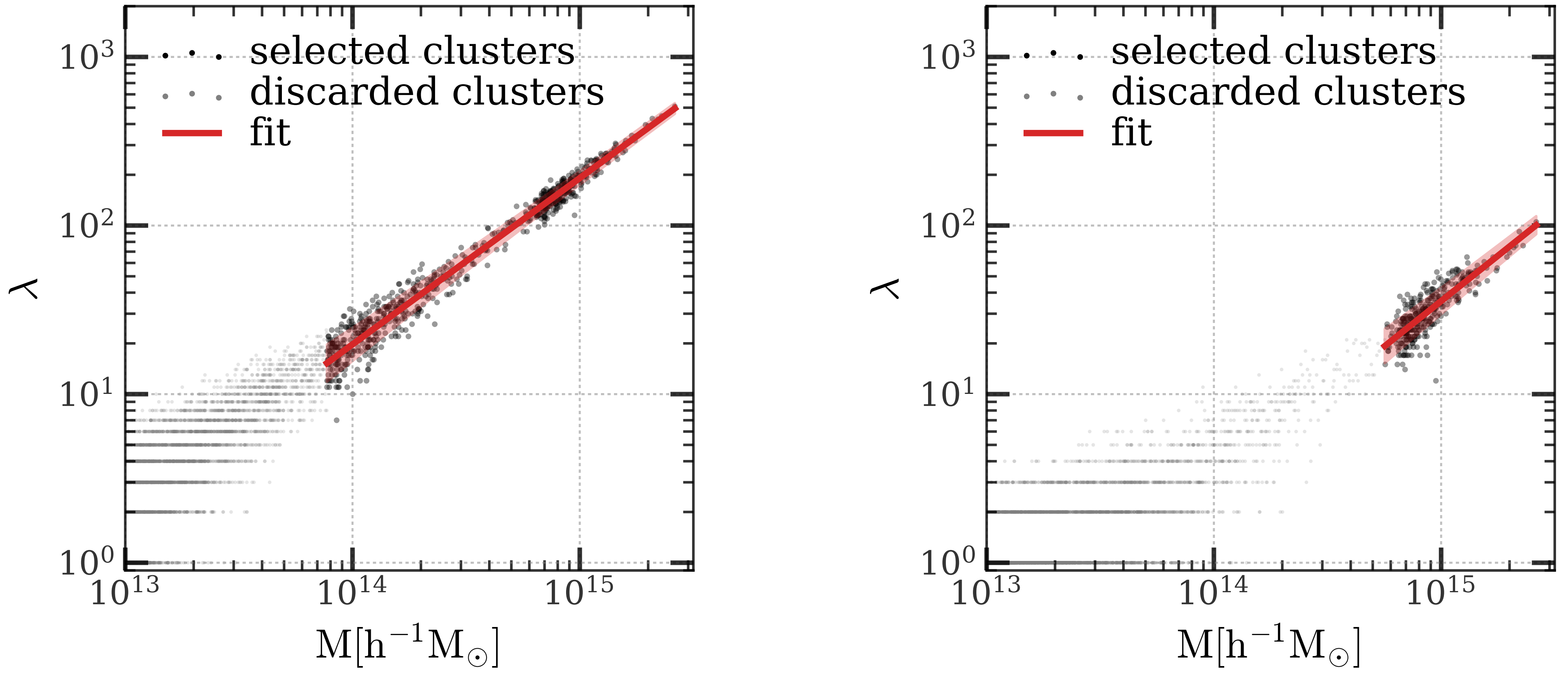
▶ 3 params $\{A, B, \sigma_I\}$

- Stellar mass range: $\log M_\star = [9.5, 10.5]$
- Redshift range: $z = [0, 0.5, 1, 1.5]$

1. σ_I is mass independent

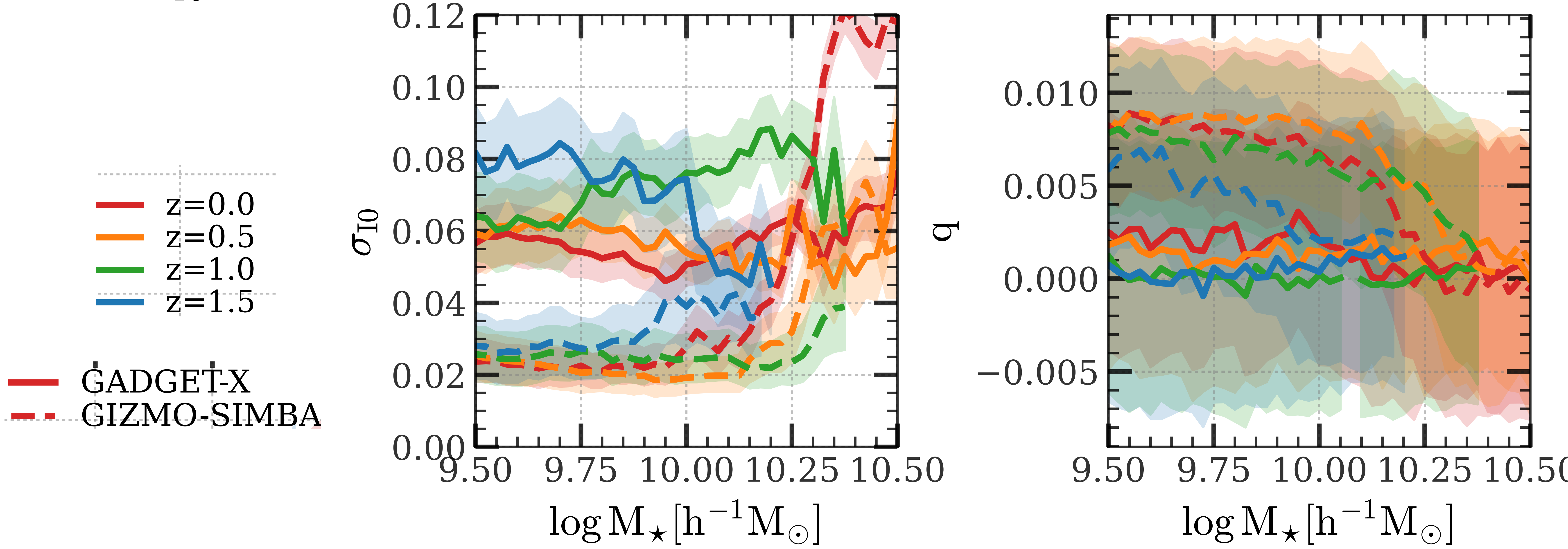
$$\sigma_I = \sigma_{I0} + q \ln \left(M/M_{piv} \right),\ 4 \text{ params}\{A, B, \sigma_{I0}, q\}$$

$$q \sim 0$$



z=0 GADGET-X $M_\star \geq 10^{9.5}\ h^{-1}M_\odot$

z=0 GADGET-X $M_\star \geq 10^{10.5}\ h^{-1}M_\odot$



Results — MR relation using $M_\star$

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Power-law: $\langle \ln \lambda \,|\, \ln M \rangle = A + B \ln \left(M/M_{piv} \right)$

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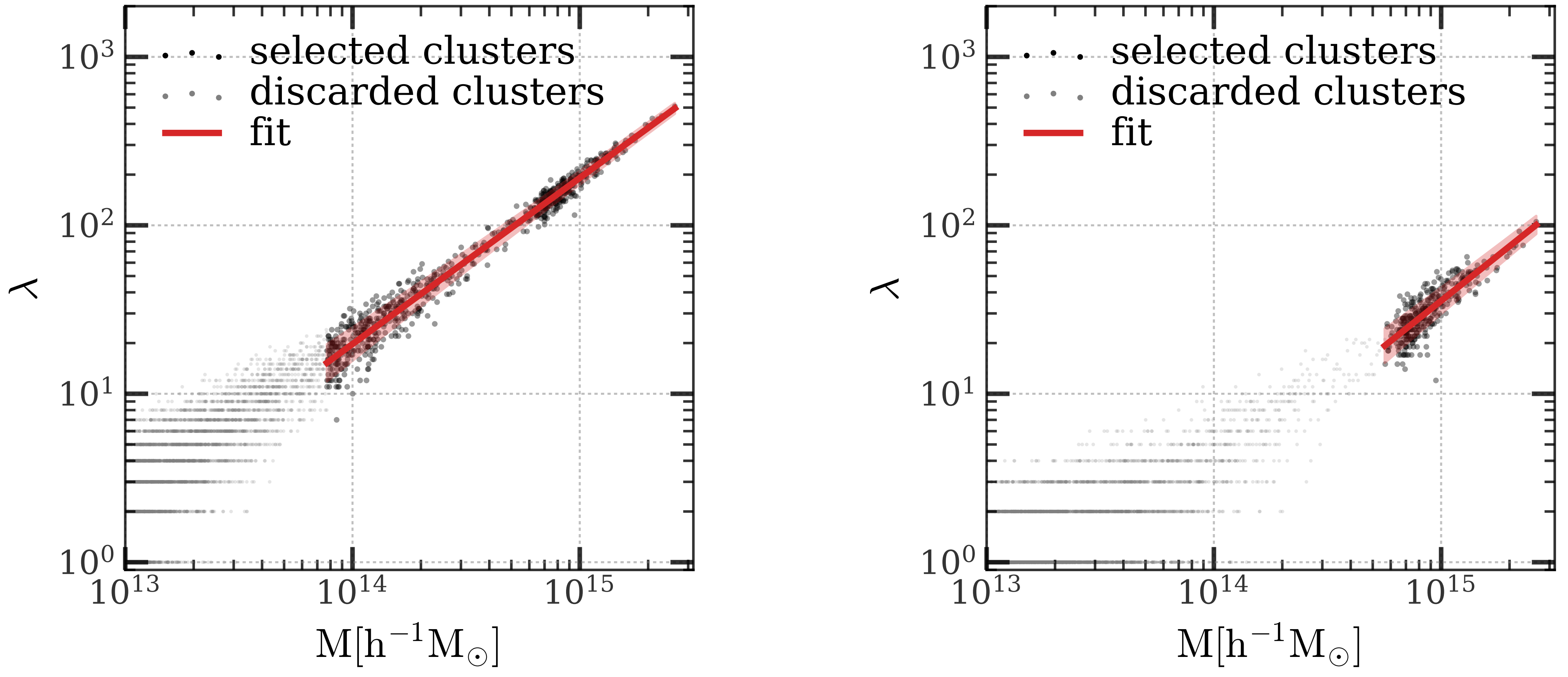


1. σ_I is mass independent
2. $M_\star \gtrsim 10^{10}\ h^{-1} M_\odot$, the behavior of parameters is influenced by the baryon models.
3. $M_\star \lesssim 10^{10}\ h^{-1} M_\odot$

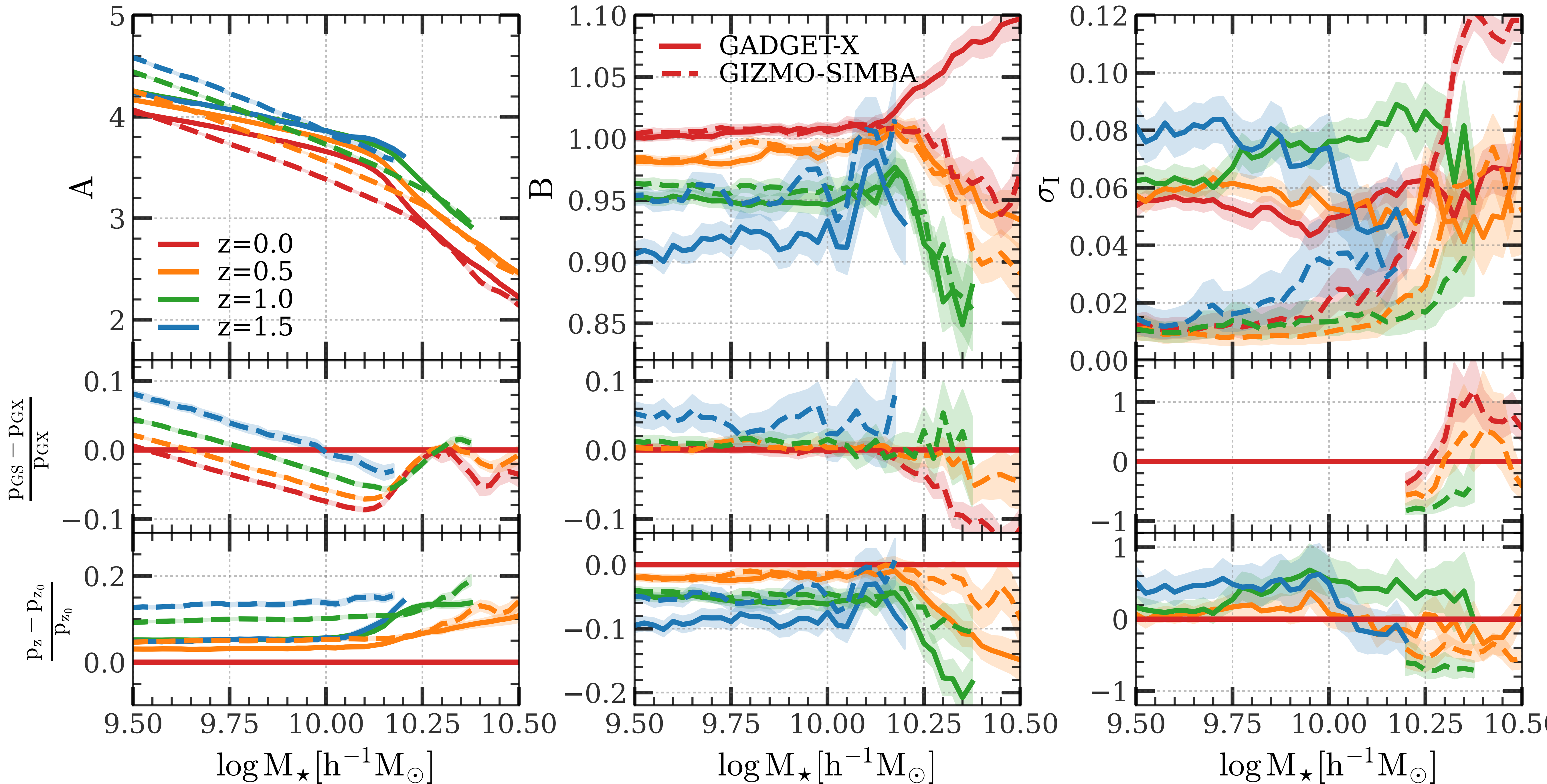
$A \to A_0 + A_z(z) \uparrow + A_\star(M_\star) \downarrow$

$B \to B_0 + B_z(z) \downarrow$

$\sigma_I \to \sigma_{I0} + \sigma_z(z) \uparrow$

σ_I is independent on M and $M_\star$ => large-scale environments.

GIZMO-SIMBA, $\sigma_I \sim 0$



Results — MR relation using $\mathscr{M}$

Whether the above results suitable for different surveys?

- Actual observations: the apparent magnitude m
  - Absolute - apparent magnitude :
    $\mathscr{M} = m - 5\log(D_L/10pc)$
- Different survey, different bands:
  - CSST i-band $\mathscr{M}_i$
  - CSST z-band $\mathscr{M}_z$
  - Euclid h-band $\mathscr{M}_h$

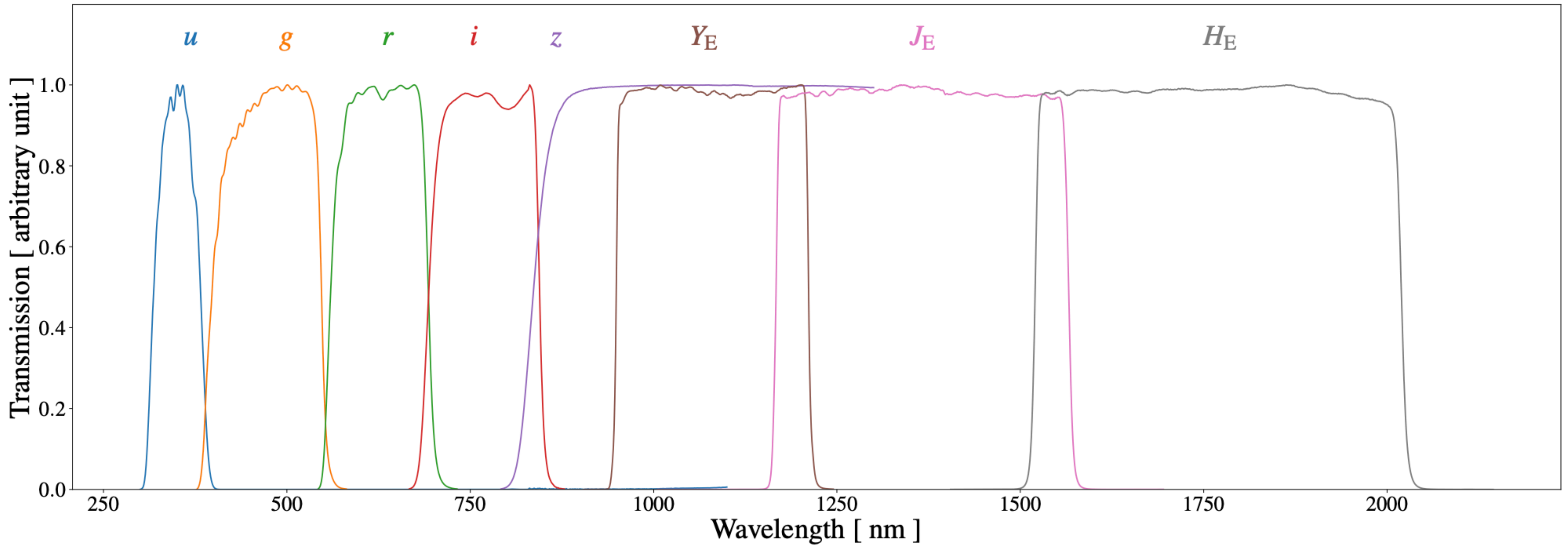
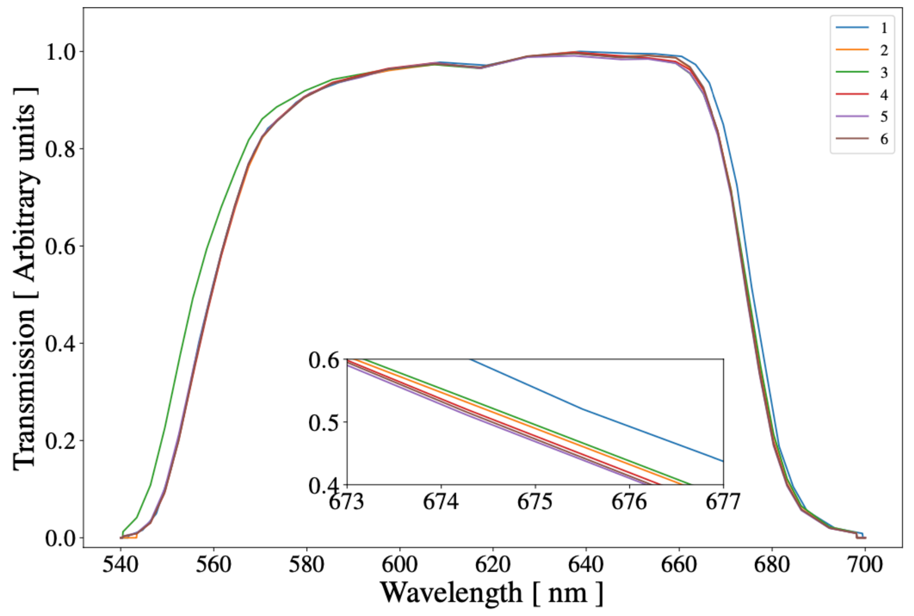
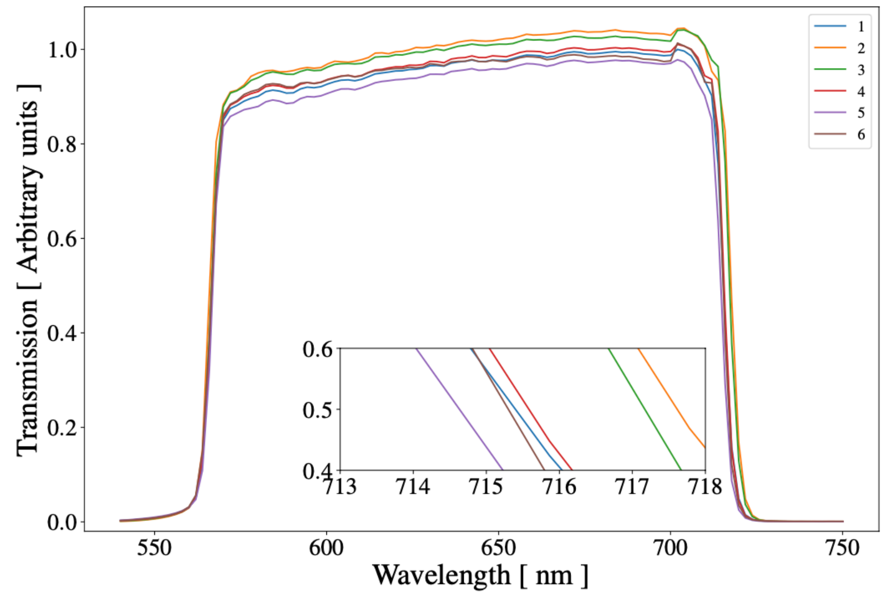


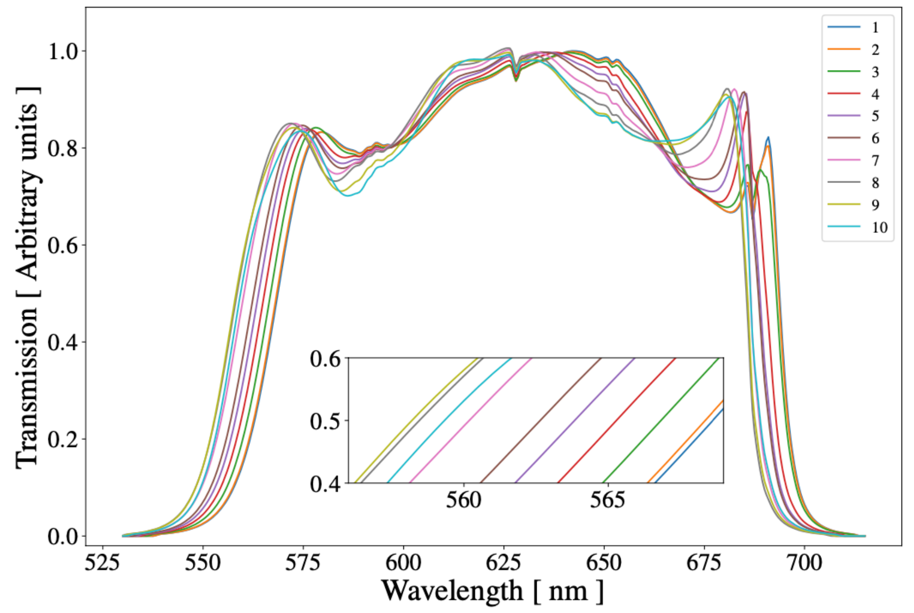
Fig. 1. Set of transmission curves $\mathcal{T} = \{ugrizY_EJ_EH_E\}$ used for the *Euclid* mission (from left to right). The *ugriz* passbands are only fiducial, since different sets will be used by *Euclid*; those represented here are from SDSS. The $Y_EJ_EH_E$ passbands are from NISP on board *Euclid*. Only the filter transmissions are shown, without atmospheric, telescope, and detector quantum efficiency effects.



r-band
SDSS



r-band
DES



r-band
MegaCam

Results — MR relation using $\mathscr{M}$

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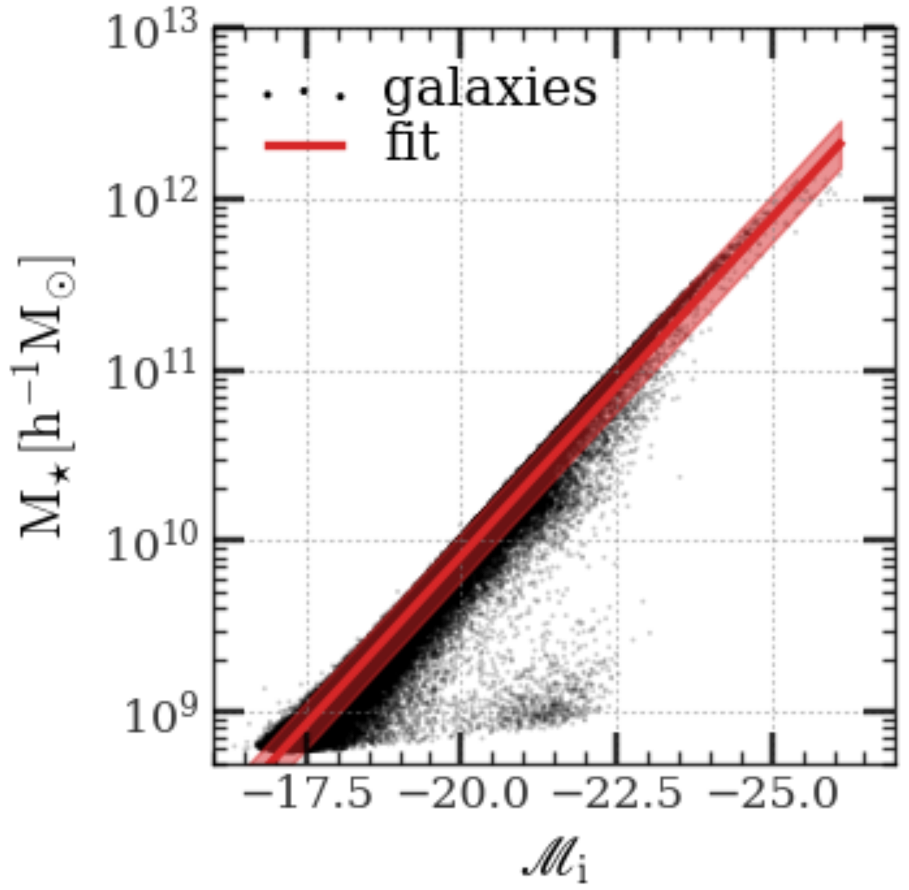
CSST i-band

1. use $\mathscr{M}_i$ to select galaxies — $\mathscr{M}_\mathrm{i}$

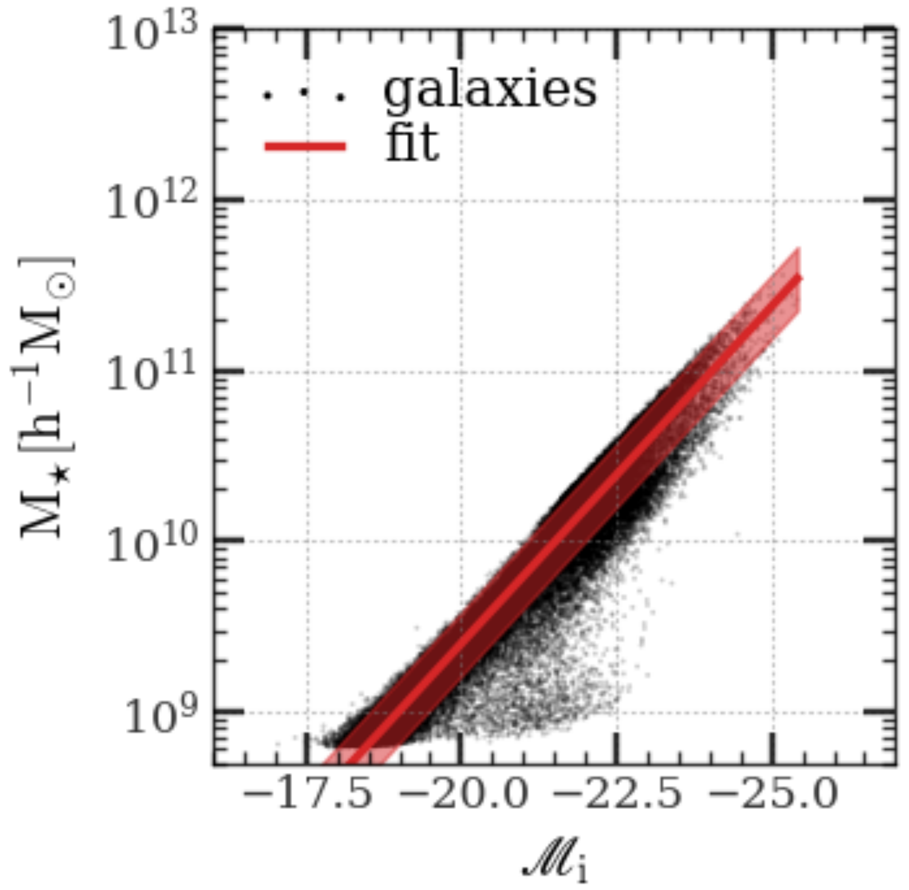
2. use $M_\star$ to select galaxies ··· $\mathrm{M}_\star$
   - Convert $\mathscr{M}_i$ to $M_\star$
   - Stellar mass-magnitude relation

$$\ln M_\star = 4.63 - 0.91\mathscr{M}_i - 1.30 \times \ln(1+z)$$

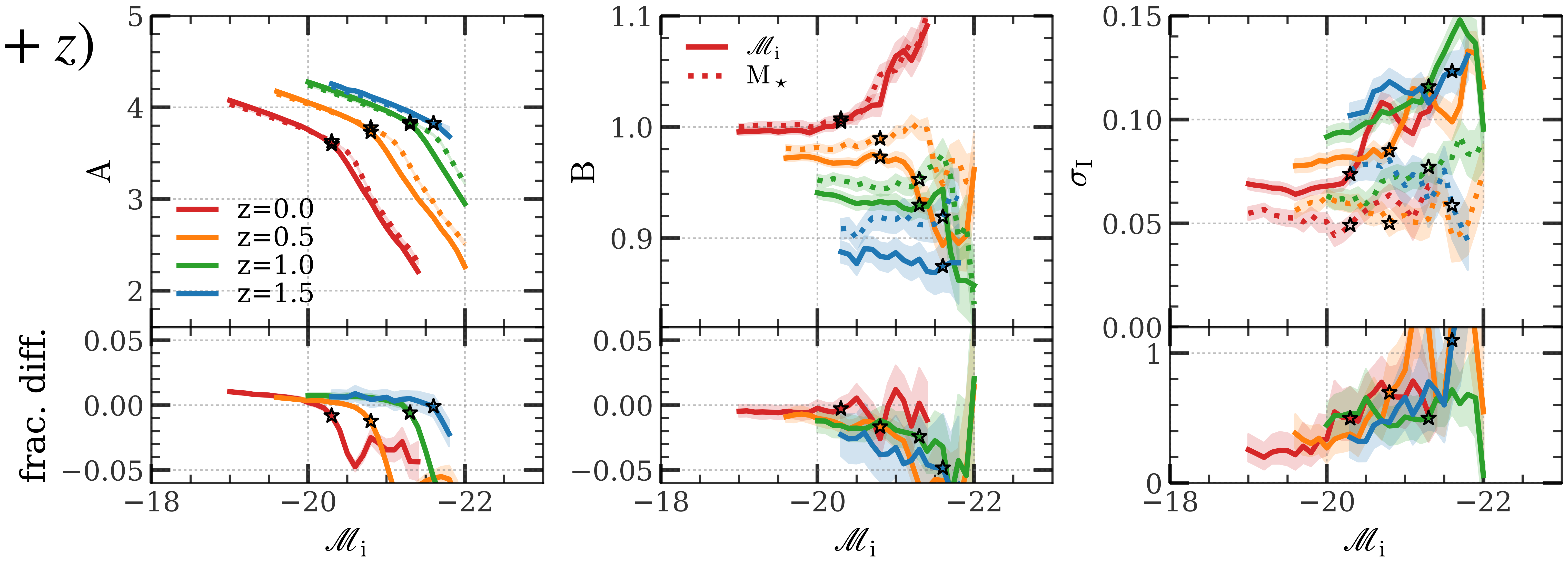
- $\{A, B\} \sim 5\%$
- $\sigma_I \times 1.5$



z=0
GADGET-X



z=1.5
GADGET-X



Results — MR relation using $\mathscr{M}$

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Other bands

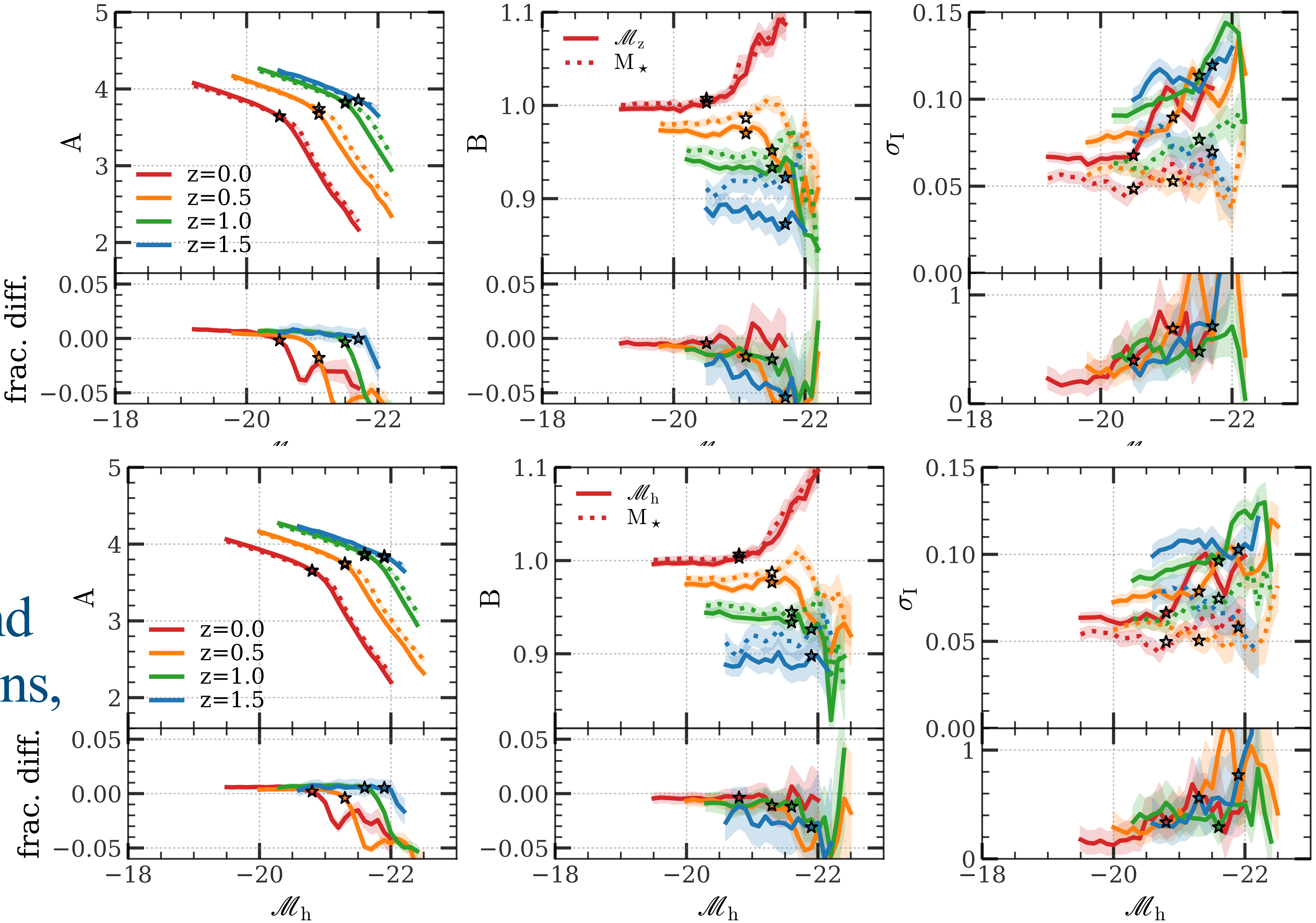
- Stellar mass-magnitude relation

$$\ln M_\star = 4.57 - 0.90\mathscr{M}_z - 1.20 \times \ln(1+z)$$

$$\ln M_\star = 4.68 - 0.88\mathscr{M}_h - 1.00 \times \ln(1+z)$$

- $\{A, B\} \sim 5\%$
- $\sigma_I \times 1.5$

We can use
(1) the MR relation selected by $M_\star$ and
(2) the different $M_\star - \mathscr{M}_{i,z,h,\ldots}$ relations,
to forecast for different surveys/bands.



5. Discussions

Discussions — richness PDF

Probability Distribution Function

☐ Log-normal

$$P(\ln\lambda \,|\, \ln M) = \frac{1}{\sqrt{2\pi}\sigma_{\ln\lambda|\ln M}} \exp\left[-\frac{(\ln\lambda - \langle \ln\lambda \,|\, \ln M\rangle)^2}{2\sigma^2_{\ln\lambda|\ln M}}\right]$$

☑ Skewed Gaussian

$$P(\lambda \,|\, M) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(\lambda - \langle \lambda^{sat} | M\rangle\right)^2}{2\sigma^2}} \operatorname{erfc}\left[-\alpha \frac{\lambda - \langle \lambda^{sat} \,|\, M\rangle}{\sqrt{2\sigma^2}}\right]$$

1. Log-normal

   1.1. Simple linear relation: $\sigma_{\ln\lambda} = \sigma_0 + q\ln\left(M/M_p\right)$

   1.2. Intrinsic scatter + Poisson term :

   $$\sigma^2_{\ln\lambda} = \sigma^2_{IG} + \frac{e^{\langle \ln\lambda\rangle} - 1}{e^{2\langle \ln\lambda\rangle}}$$

2. Skewed Gaussian

   2.1. Intrinsic scatter: σ_I

Discussions — richness PDF

Probability Distribution Function

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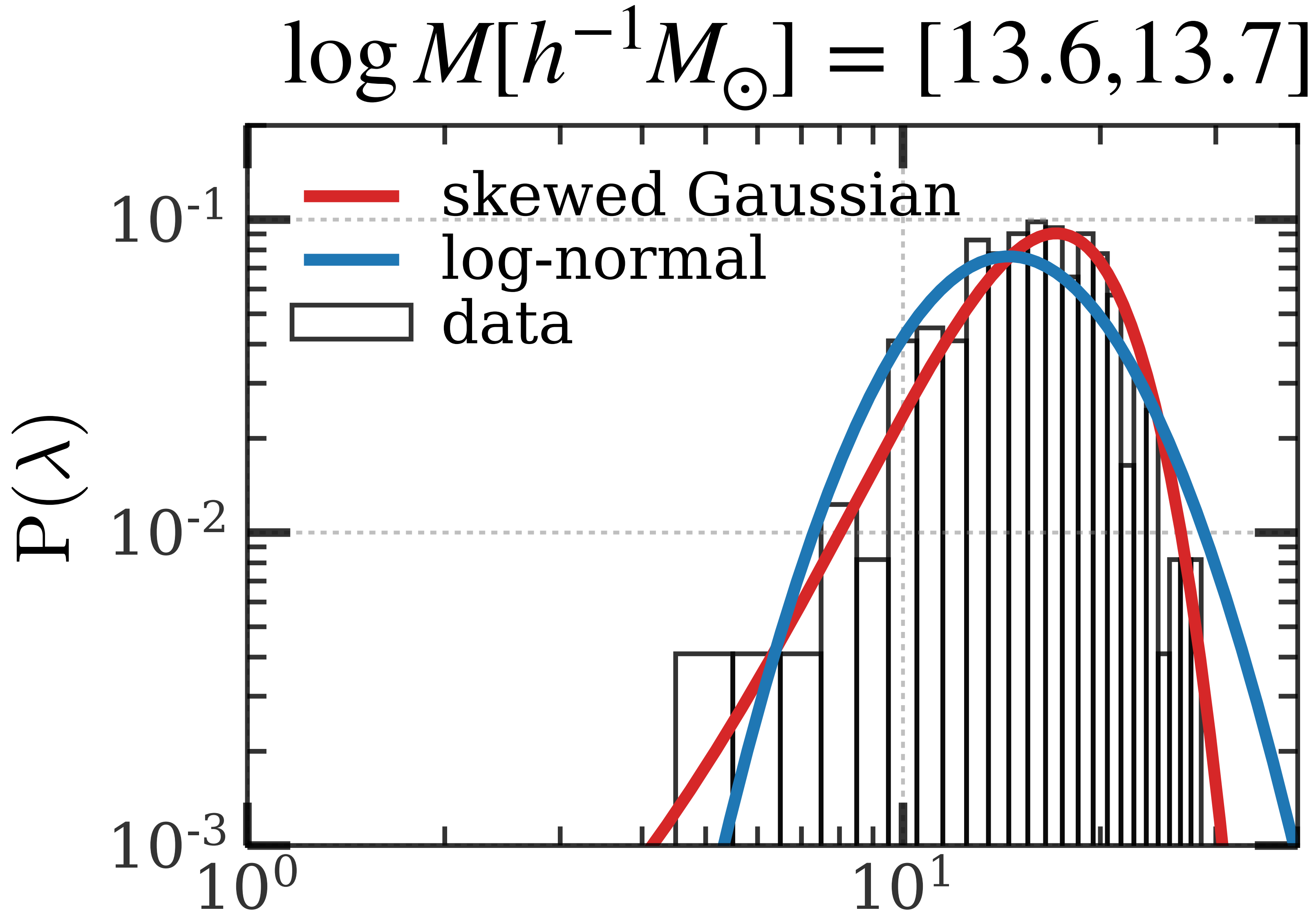
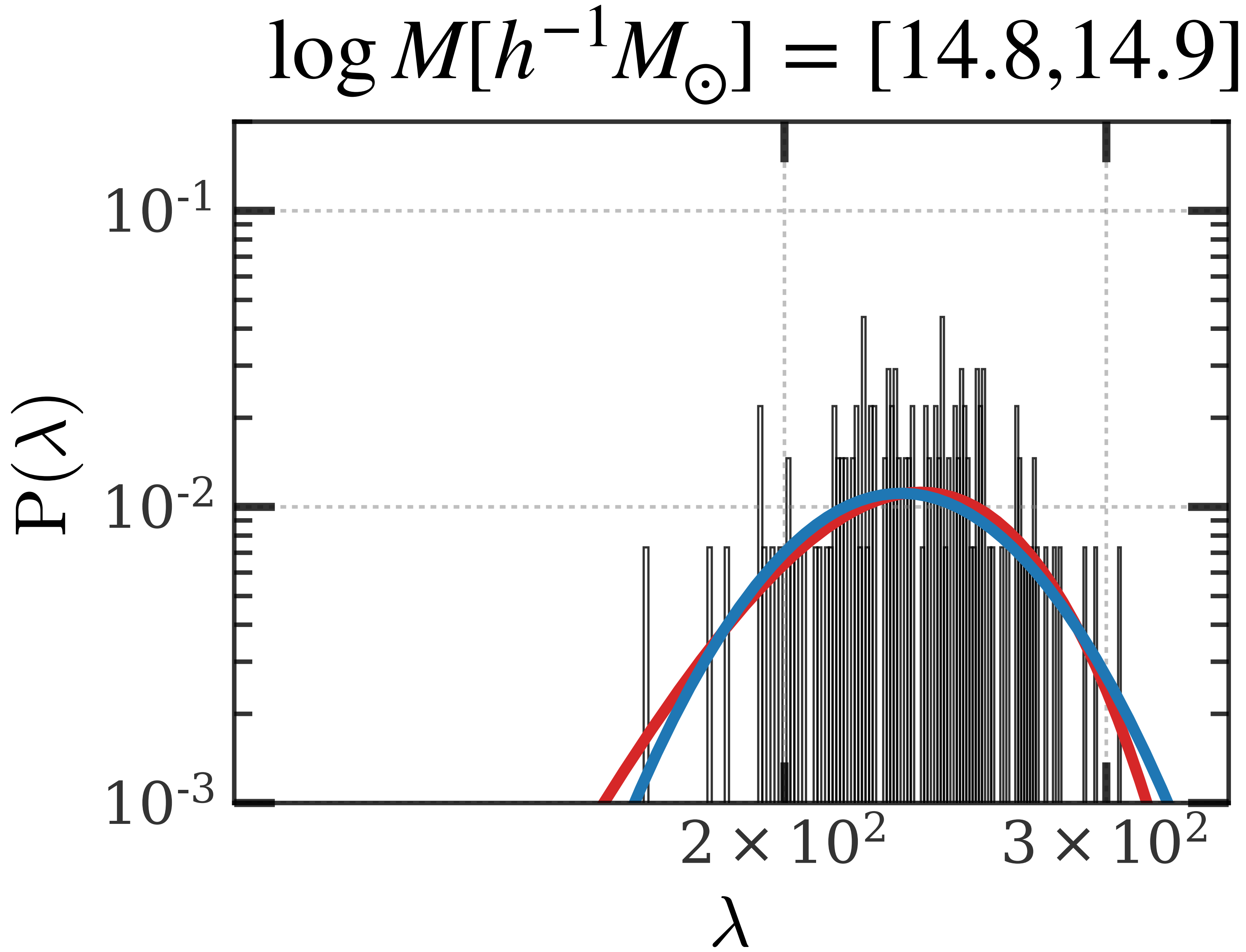
1. Log-normal v.s. Skewed Gaussian

- $\log M[h^{-1}M_{\odot}] = [13.6, 13.7]$
  - Residual: 9.96>5.34
  - the skewed Gaussian function better incorporates low-richness values



- $\log M[h^{-1}M_{\odot}] = [14.8, 14.9]$
  - Residual: 31.0>29.9
  - the two functions exhibit greater consistency in the larger mass bin



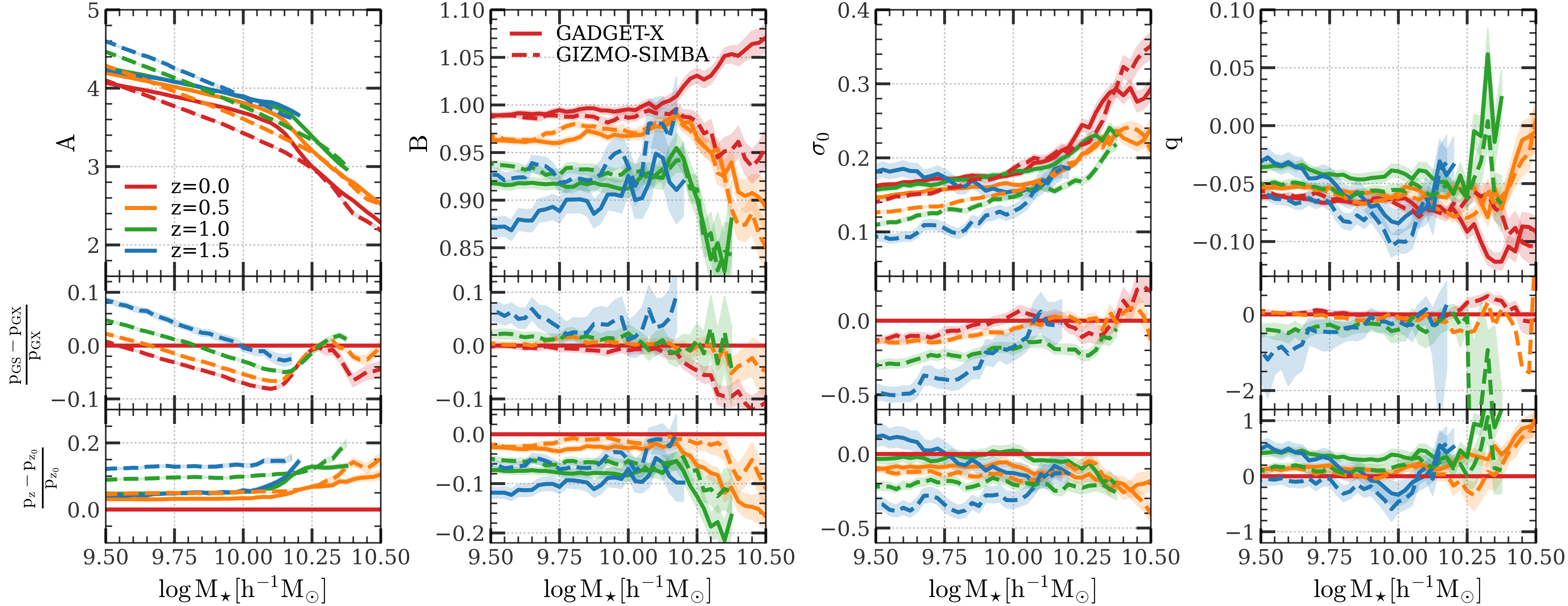
Discussions — richness PDF

Probability Distribution Function

University of Science and Technology of China

1.1 Log-normal: $\sigma_{\ln \lambda} = \sigma_0 + q \ln \left(M/M_p \right)$

- $\{A, B, \sigma_0, q\}$ v.s. $\{A, B, \sigma_I\}$ — more parameters
- $\sigma_0(M_\star)$ — intricate scatter



Discussions — richness PDF

Probability Distribution Function

University of Science and Technology of China

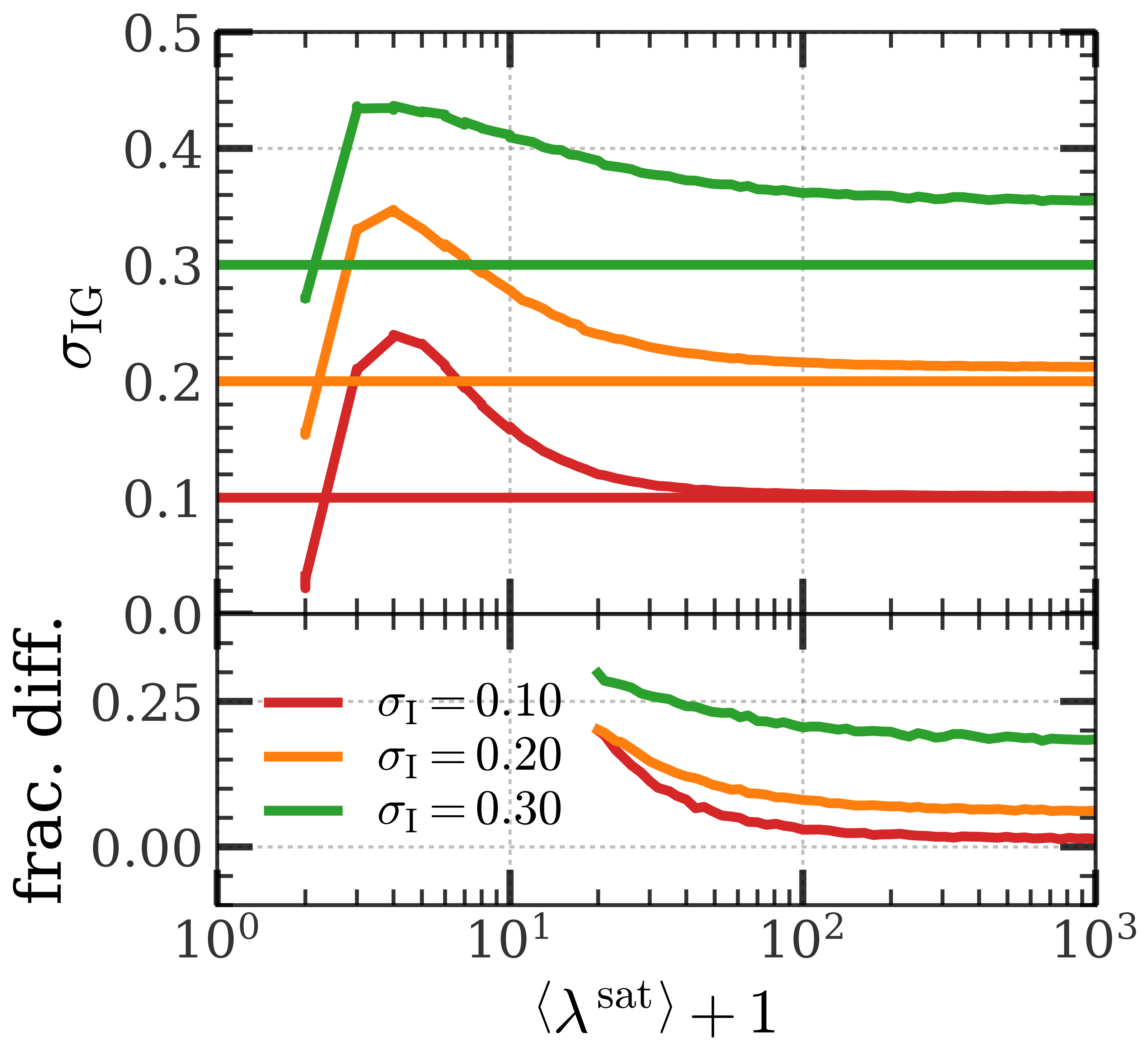
1.2 Log-normal: $\sigma^2_{\ln\lambda} = \sigma^2_{IG} + \frac{e^{\langle \ln\lambda \rangle} - 1}{e^{2\langle \ln\lambda \rangle}}$

- motivated by the super-Poisson distribution in the HOD model
  - σ_I is mass-independent
  - $\sigma_{IG}(M)$?

1. Sampling:
   - select $\langle \lambda^{sat} \rangle, \sigma_I$ => sample 10^6 λ => calculate $\sigma_{\ln\lambda}, \langle \ln\lambda \rangle$

=> calculate $\sigma^2_{IG} = \sigma^2_{\ln\lambda} - \frac{e^{\langle \ln\lambda \rangle} - 1}{e^{2\langle \ln\lambda \rangle}}$

   - σ_{IG} is mass-dependent
   - $\sigma_{IG} > \sigma_I$



Sampling

Discussions — richness PDF

Probability Distribution Function

University of Science and Technology of China

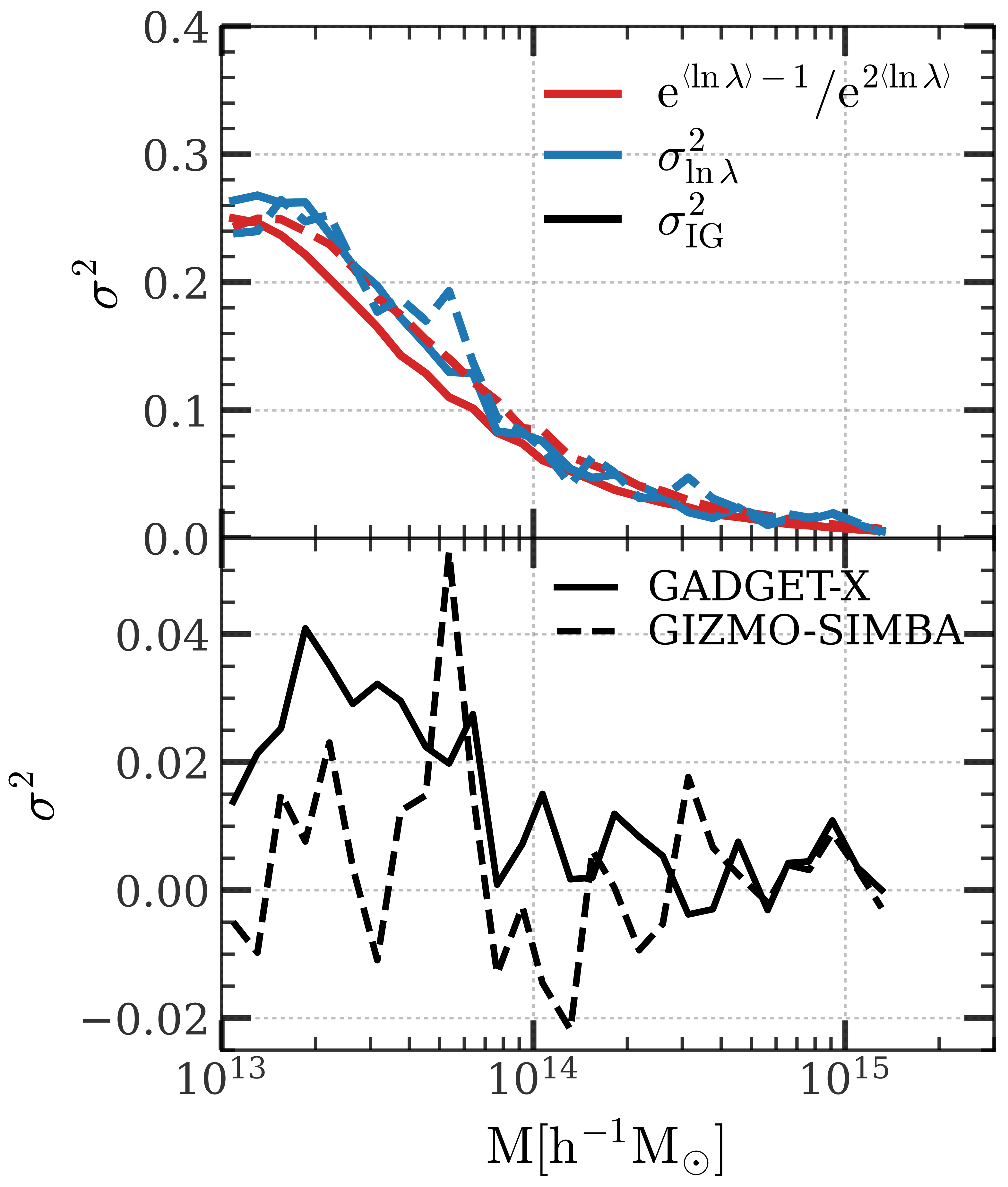
1.2 Log-normal: $\sigma^2_{\ln\lambda} = \sigma^2_{IG} + \frac{e^{\langle \ln\lambda \rangle} - 1}{e^{2\langle \ln\lambda \rangle}}$

- motivated by the super-Poisson distribution in the HOD model
  - σ_I is mass-independent
  - $\sigma_{IG}(M)$?

1. Sampling:
   - σ_{IG} is mass-dependent, $\sigma_{IG} > \sigma_I$
2. Data:
   - select $[M, M + \Delta M]$ => a set of λ => calculate $\sigma_{\ln\lambda}$, $\langle \ln\lambda \rangle$

=> calculate $\sigma^2_{IG} = \sigma^2_{\ln\lambda} - \frac{e^{\langle \ln\lambda \rangle} - 1}{e^{2\langle \ln\lambda \rangle}}$

- GIZMO-SIMBA: $\sigma_{IG} \sim 0$
- GADGET-X: σ_{IG} is mass dependent, $\sigma_{IG} > \sigma_I$



Data

Discussions — richness PDF

Probability Distribution Function

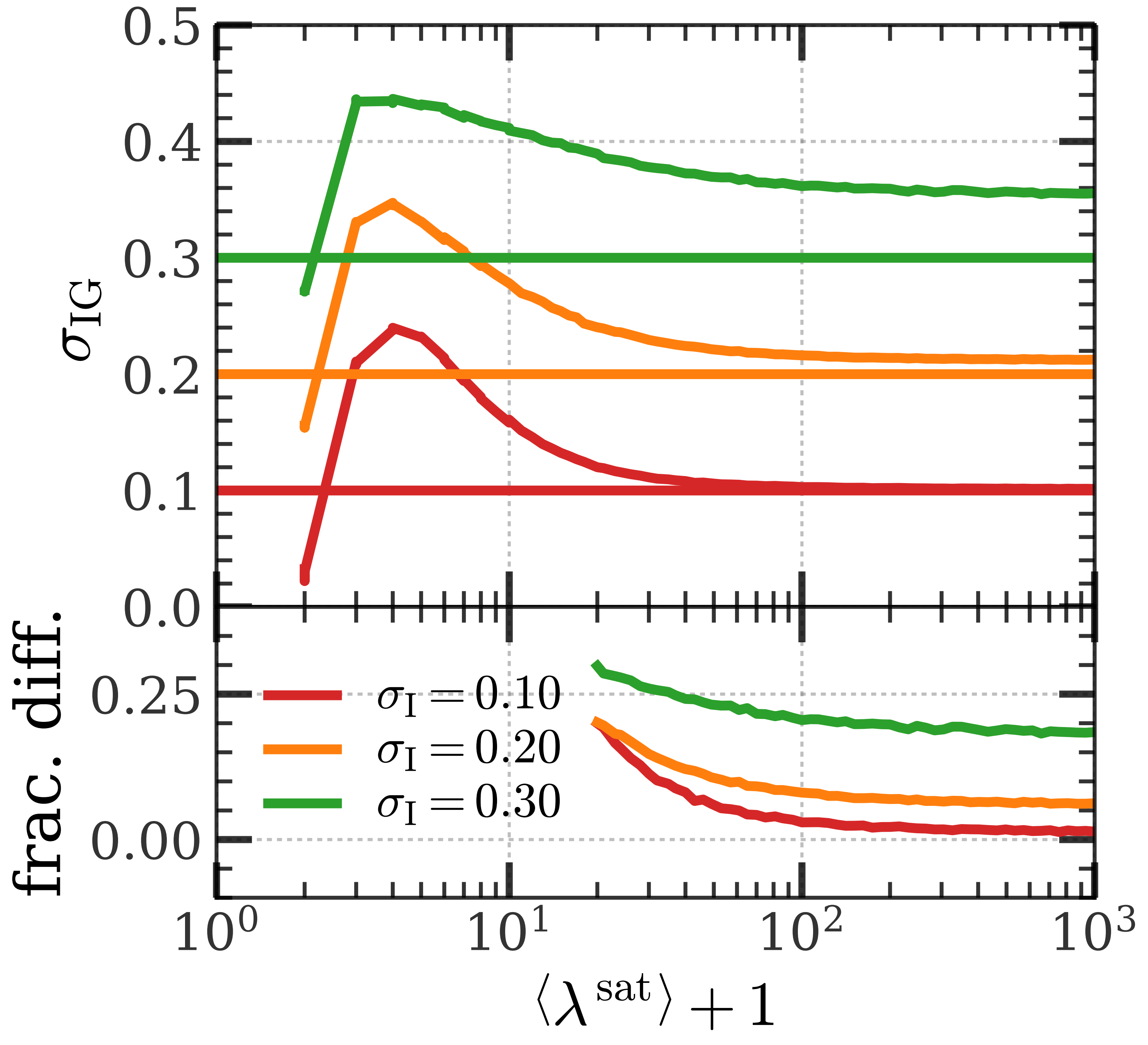
University of Science and Technology of China

1.2 Log-normal: $\sigma^2_{\ln\lambda} = \sigma^2_{IG} + \dfrac{e^{\langle \ln\lambda\rangle} - 1}{e^{2\langle \ln\lambda\rangle}}$

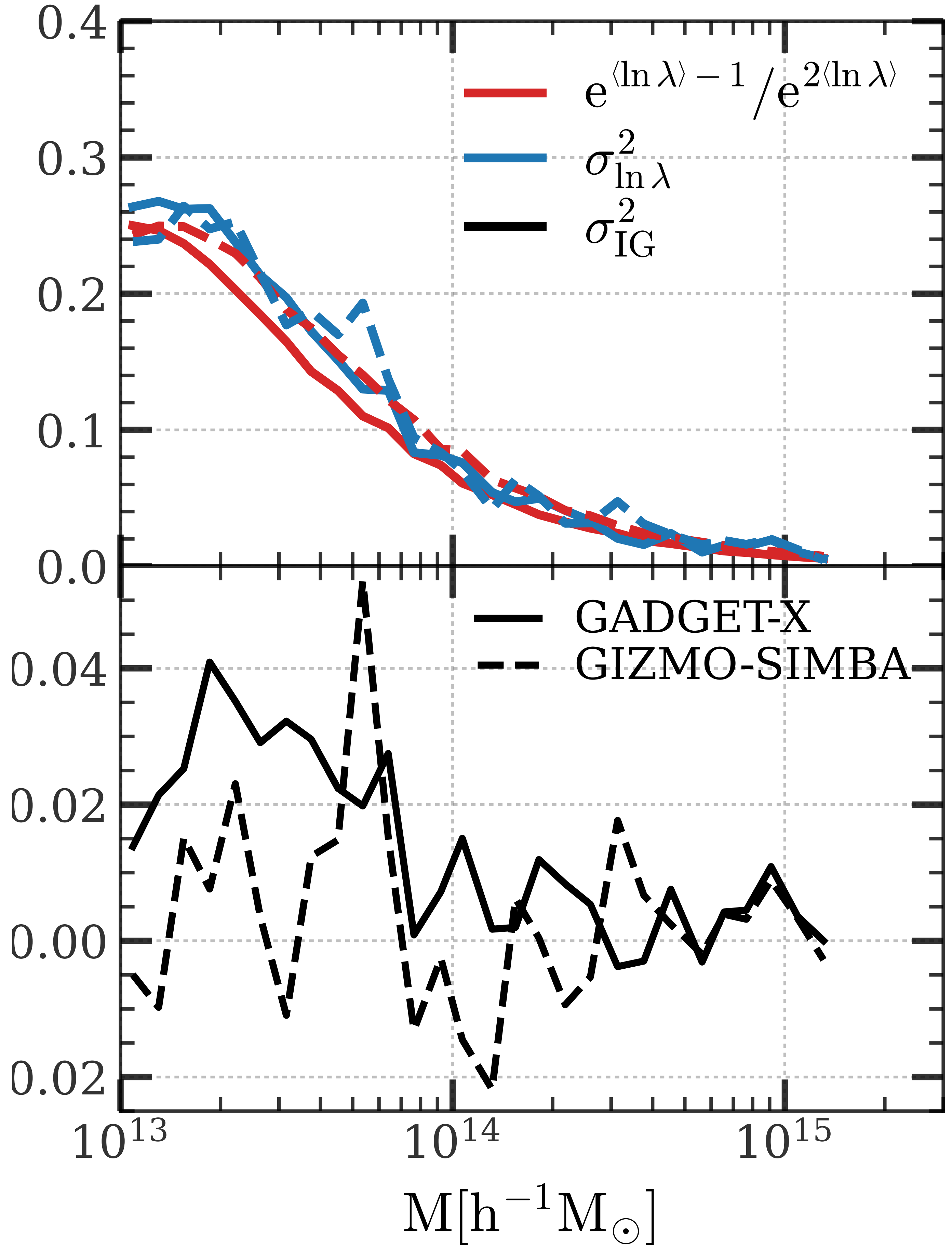
- motivated by the super-Poisson distribution in the HOD model
  - σ_I is mass-independent
  - $\sigma_{IG}(M)$?

σ_{IG} is mass-dependent.
Overlook this dependence will overestimate the scatter.

To compare with other papers, we overlook it.



Sampling



Data

Discussions — 7-parameters

中国科学技术大学
University of Science and Technology of China

3 params -> 7

- Skew Gaussian

$$A \to A_0 + A_z \times \ln\frac{1+z}{1+z_p} + A_\star \times \ln\frac{M_\star}{M_{\star p}}$$
$$B \to B_0 + B_z \times \ln\frac{1+z}{1+z_p}$$
$$\sigma_I \to \sigma_{I0} + \sigma_z \times \ln\frac{1+z}{1+z_p},$$

- Log-normal

$$A \to A_0 + A_z \times \ln\frac{1+z}{1+z_p} + A_\star \times \ln\frac{M_\star}{M_{\star p}}$$
$$B \to B_0 + B_z \times \ln\frac{1+z}{1+z_p}$$
$$\sigma_{IG} \to \sigma_{IG0} + \sigma_z \times \ln\frac{1+z}{1+z_p},$$

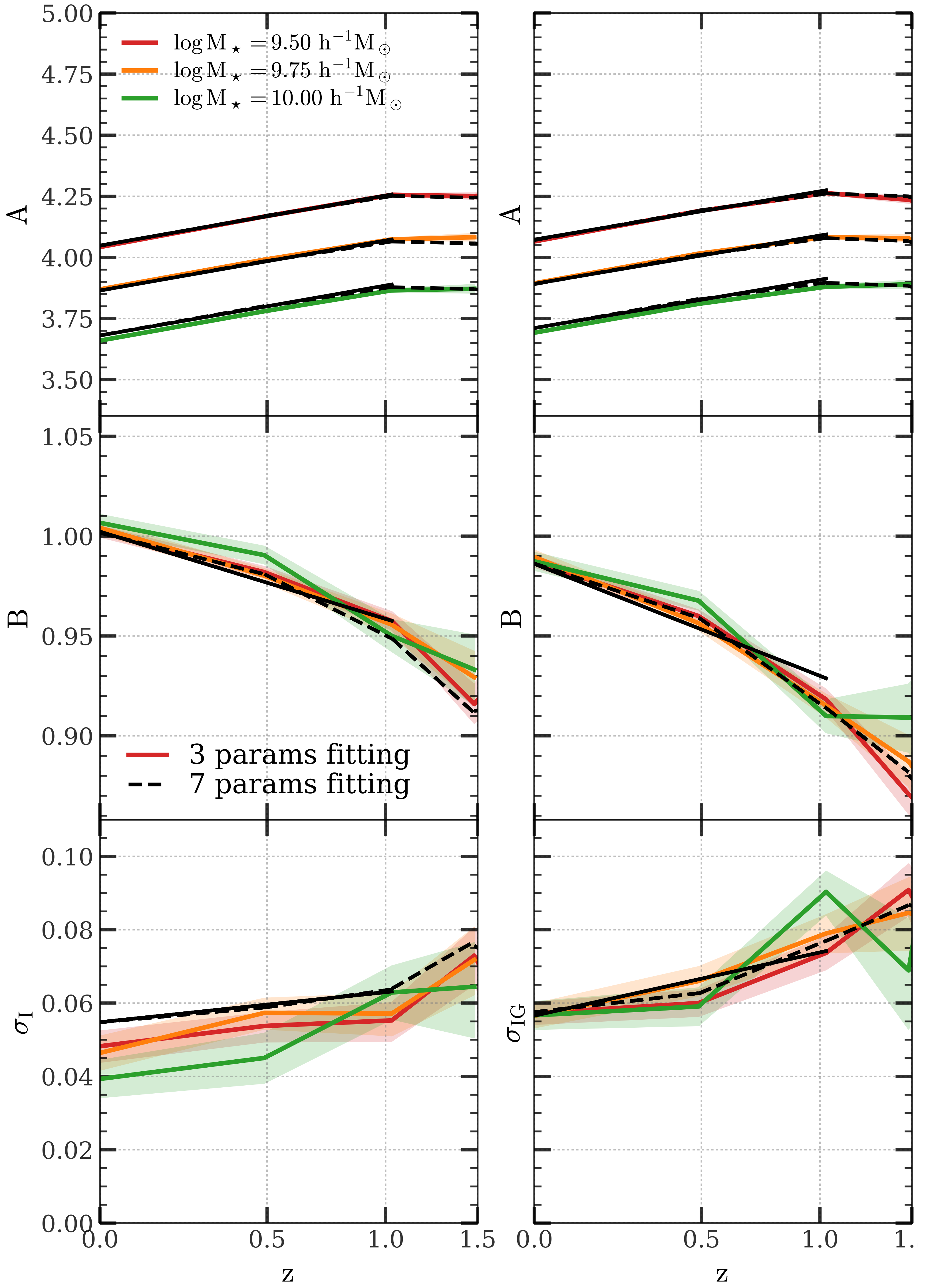


Table 1. The 7 fitting parameters for GADGET-X. The upper panel displays the results obtained using the skewed Gaussian distribution, while the lower panel shows the results obtained using the log-normal distribution. Each column corresponds to a different redshift range. Fitting errors smaller than 10% have been omitted for a cleaner presentation.

z	[0,1]	[0,0.5]	[0.5,1]	[1,1.5]
A_0	3.792	3.803	3.800	3.887
A_z	0.205	0.245	0.150	$-0.017^{+0.006}_{-0.006}$
$A_\star$	-0.320	-0.319	-0.323	-0.325
B_0	0.980	0.981	0.980	0.993
B_z	-0.031	-0.042	-0.060	-0.083
σ_{I0}	0.060	0.059	0.059	0.048
σ_z	$0.008^{+0.001}_{-0.001}$	$0.008^{+0.002}_{-0.002}$	$0.009^{+0.002}_{-0.002}$	$0.029^{+0.004}_{-0.004}$
A_0	3.819	3.833	3.829	3.911
A_z	0.196	0.244	0.128	$-0.028^{+0.006}_{-0.007}$
$A_\star$	-0.314	-0.313	-0.316	-0.318
B_0	0.957	0.959	0.958	0.952
B_z	-0.044	-0.056	-0.084	-0.072
σ_{IG0}	0.067	0.063	0.063	0.066
σ_z	0.019	$0.011^{+0.002}_{-0.002}$	0.026	$0.021^{+0.005}_{-0.004}$

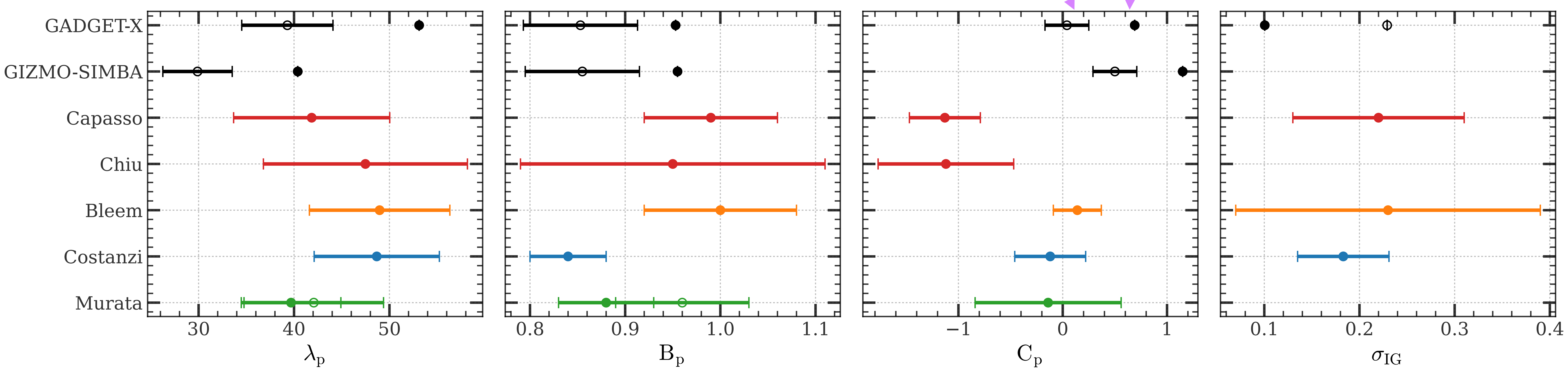
Discussions —— comparison with previous work

中国科学技术大学 University of Science and Technology of China

1. Apparent - absolute magnitude : $\mathscr{M}_i = m_i - 5\log(D_L/10pc)$
2. Stellar mass - abs magnitude: $\ln M_\star = 4.63 - 0.91\mathscr{M}_i - 1.30 \times \ln(1+z)$
3. Stellar mass threshold result: 7 parameters $\{A_0, A_z, A_\star, B_0, B_z, \sigma_{IG0}, \sigma_z\}$

redMaPPer $0.2L_* \Rightarrow m_i = 22, \mathscr{M}_i = -21.29$

Red fraction $f_{red} \propto (1+z)^{0.65\pm0.21}$



Capasso 2019: X-ray, ROSAT, galaxy dynamics
Chiu 2023: X-ray, eROSITA, number counts(NC)
Bleem 2020: SZ, SPT, NC
Costanzi 2021: optical, DES, NC
Murata 2019: optical, HSC, NC

Pivot point $M = 3e14, z = 0.5$

Mass dependence M^B

Redshift dependence $(1+z)^C$

6. Conclusions

Conclusions

- Skew Gaussian function has a simpler and smaller scatter σ_I than others.
  - σ_I is independent on M and $M_\star$ => large-scale environments.
  - GIZMO-SIMBA has a negligible σ_I.
- When $M_\star \gtrsim 10^{10} h^{-1} M_\odot$, MR relation strongly depends on the baryon models.
- When $M_\star \lesssim 10^{10} h^{-1} M_\odot$, MR relation can be fitted with 7 parameters.
- Apply MR relation selected by $M_\star$ to different surveys/bands, once we know the $M_\star - \mathscr{M}$ relation.