Modified gravity and its lensing constraint

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01 Modified gravity

- General relativity (GR) as a basic theory of gravity, its combination with cold dark matter theory gives the Lambda CDM model, which has been well verified in today's cosmological and galaxy dynamics observations.
- Hubble tension
- Lack of observational evidence for the particle physics explanation
- Theoretical challenges to cosmological constant vacuum energy
- Additionally, many emerging observational fields, such as gravitational waves, black holes, etc., provide a strong impetus for considering new ways to construct gravitational theories different from GR.
- Modified gravity

- Two commonly used formalisms of modified gravity:
- Curvature formalism:
- Extend GR on a geometric basis:
  - Extend GR on a geometric basis.Theories of gravity with rich geometric structures have been proposed, such as f(R) theory.
- Equivalent torsional (teleparallel) formalism :
  - Extension of teleparallel gravity (TEGR): which is based on the Riemann-Catan space-time and has an asymmetric Weitzenbock connection.
  - Unlike the Levy-Chevita connection of GR, Weitzenbock connection produces torsion but it is curvature-free.
  - In TEGR, torsion plays the role of curvature, and the tetrad field plays the role of dynamic field rather than metric.

Teleparallel Equivalent of General gravity (TEGR):

- The relation between tetrad and the manifold metric is

$$g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu$$

- The teleparallel connection:

$$\Gamma^\sigma_{\nu\mu} := h_A{}^\sigma \partial_\mu h^A{}_\nu + h_A{}^\sigma \omega^A{}_{D\mu} h^D{}_\nu$$

Spin metric-compatible connection :

$$\omega^A{}_{D\mu} = \Lambda^A{}_C\, \partial_\mu (\Lambda^{-1})^C{}_D$$

- The torsion tensor and torsion scalar:

$$T^\lambda{}_{\mu\nu} = h_a{}^\lambda (\partial_\mu h^a{}_\nu - \partial_\nu h^a{}_\mu + \omega^a{}_{b\mu} h^b{}_\nu - \omega^a{}_{b\nu} h^b{}_\mu)$$

$$T = \frac{1}{2} S^{\alpha\mu\nu} T_{\alpha\mu\nu} = \frac{1}{4} T^{\mu\nu\rho} T_{\mu\nu\rho} + \frac{1}{2} T^{\mu\nu\rho} T_{\rho\nu\mu} - T_\rho T^\rho$$

- Here the superpotential tensor is:

$$S_\rho{}^{\mu\nu} \equiv \frac{1}{2}\left(K^{\mu,\nu}{}_\rho + \delta^\mu_\rho T^{\alpha\nu}{}_\alpha - \delta^\nu_\rho T^{\alpha\mu}{}_\alpha\right)$$

- The action and the Lagrangian(independent with spin connection, spin connection contributes only to the boundary term) of TEGR:

$$S = \int d^4x \frac{h}{16\pi G} [T + \mathcal{L}_m]$$

$$\mathcal{L}\left(h^a{}_\mu, \omega^a{}_{b\mu}\right) = \mathcal{L}\left(h^a{}_\mu, 0\right) + \frac{1}{\kappa} \partial_\mu \left(h\omega^\mu\right)$$

Spherically symmetric solutions

- In the case of low-redshift Universe and weak gravitational fields approximation, any deviation from GR can be quantified as:

$$f(T) = -2\Lambda + T + \alpha T^2 + \mathcal{O}(T^3)$$

- The spherically symmetric solutions

$$ds^2 = c^2 e^{A(r)} dt^2 - e^{B(r)} dr^2 - r^2 d\Omega$$

$$A(r) = -\frac{2GM}{c^2 r} - \frac{\Lambda}{3} r^2 - \frac{32\alpha}{r^2}$$

$$B(r) = \frac{2GM}{c^2 r} + \frac{\Lambda}{3} r^2 + \frac{96\alpha}{r^2},$$

- Corresponding to a gravitational potential that deviates from Newtonian gravity:

$$\Phi(\vec{\xi}, z) = \Phi_{\mathrm{Newton}} - 20\frac{\alpha c^2}{r^2}$$

New test on general relativity and $f(T)$ torsional gravity from galaxy-galaxy weak lensing surveys

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- The effective lensing potential :

$$\psi(\vec{\xi}) = \frac{D_{ds}}{D_d D_s} \frac{2}{c^2} \int \Phi(\vec{\xi}, z) dz$$

- The lensing convergence and effective surface mass density:

$$\kappa = \frac{4\pi G}{c^2} \frac{D_d D_{ds}}{D_s} \left[\Sigma(R) - \frac{10\alpha c^2}{GR^3} \right]$$

$$\Sigma_{\mathrm{eff}} = \Sigma - \frac{10\alpha c^2}{GR^3}$$

- Fitting ESD:

Consider Rc = R1/2, where R1/2 ≈ 0.015R200 is the radius that encloses half of stellar mass. Defining ε ≡ Rc/R, the modified ESD profile takes the form:

$$\Delta\Sigma_{\text{eff}}(R) = \Delta\Sigma(R) + \frac{5c^4\Omega_\alpha^0(2-\epsilon-\epsilon^2)}{9GH_0^2R^3\epsilon(1+\epsilon)} \qquad \alpha = c^2\Omega_\alpha^0/(18H_0^2)$$

Use a simple NFW with α and treat halo mass and concentration as free parameters .

Adopt the concentration-mass relation as a Gaussian prior to suppress the contribution from α, which tests the lower limit of the upper bound .

Allow α to vary in different mass bins and adopt off-center effect to further suppress the contribution from NFW halo on small scales, which tests the upper limit of the upper bound.

- Data component:

lens: group catalog based on SDSS DR7

source: SDSS DR7 shear catalog

- **Result:**

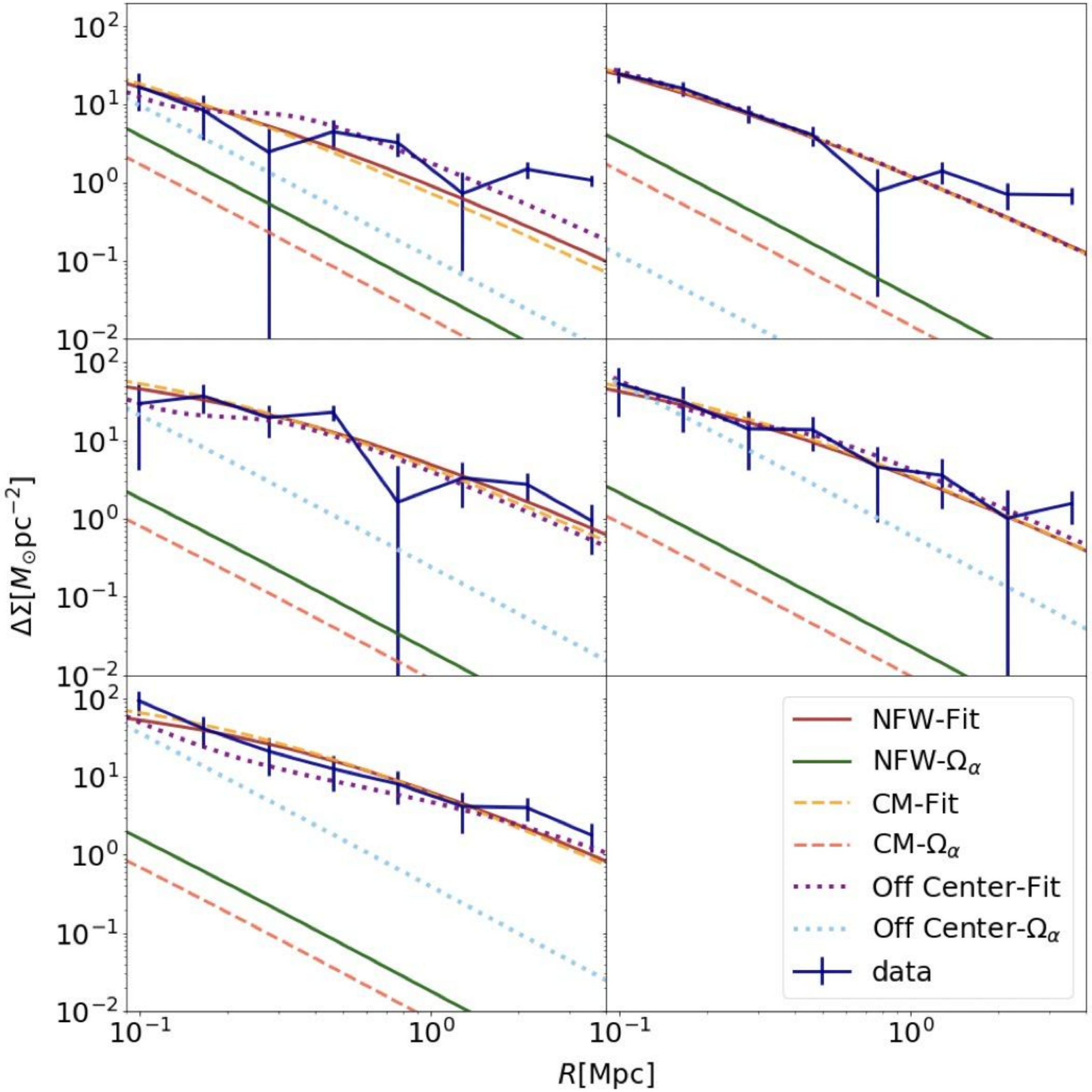
- Properties of the lens samples:
-

$\log_{10} M_{st}$ range	N_{sat}	$\langle z \rangle$	$\langle \log_{10} M_{st} \rangle$	$\langle \log_{10} M_h \rangle$
8.5-10.5	145 298	0.091	10.266	11.995
10.5-10.8	104 773	0.123	10.648	12.441
10.8-10.9	28 833	0.143	10.848	12.748
10.9-11.0	22 427	0.155	10.946	12.922
11.0-11.8	24 841	0.165	11.087	13.237

- Theoretical systematics are estimated with “CM-” and “Off Center-”, with “CM-” provide a lower limit and the fourth stellar mass bin of “Off Center-” fit provide an upper limit

$$\alpha \leq 0.33^{+1.76}_{-0.21} \ \mathrm{pc}^2 \ [\frac{R_c}{0.015 R_{200}}]$$

$$\log_{10}\Omega_\alpha \leq -18.52^{+0.80}_{-0.42} \ [\frac{R_c}{0.015 R_{200}}]$$



- Another Spherically symmetric solutions

Deflection angle and lensing signature of covariant $f(T)$ gravity

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Desire to obtain the spherically symmetric metric like: $ds^2 = A(r)^2dt^2 - B(r)^2dr^2 - r^2d\Omega^2$

perturbative solution with a small deviations form TEGR: $f(T) = T + \alpha T^2,$

$$A(r)^2 = 1 - \frac{2M}{r} + \epsilon a(r)\ , \qquad a(r) \approx -\frac{8\alpha M^3}{5r^5} + \mathcal{O}\left(\frac{1}{r^6}\right)\ ,$$

$$B(r)^2 = \left(1 - \frac{2M}{r}\right)^{-1} + \epsilon b(r) \qquad b(r) \approx \frac{8\alpha M^3}{r^5} + \mathcal{O}\left(\frac{1}{r^6}\right)$$

The deviation of the image position and the total magnification:

$$\Delta\theta \approx \theta_{f(T)} - \theta_{GR} = \frac{32\alpha}{15\theta_0^3\left(\theta_0^2+1\right)M^2}\varepsilon^4 + \mathcal{O}(\varepsilon^5)\ .$$

$$\Delta\mu_{\text{tot}} = \mu_{\text{tot}\,f(T)} - \mu_{\text{tot}\,GR} \approx -\frac{64\left(\beta^4+6\beta^2+6\right)\alpha}{15\beta\left(\beta^2+4\right)^{3/2}M^2}\varepsilon^4 + \mathcal{O}(\varepsilon^5)$$

Another Spherically symmetric solutions

The tetrad inTEGR is a free amount with no observed effect, one can operate in imaginary space

$$h^A_{(2)\mu} = \begin{pmatrix} 0 & i\mathcal{B}(r) & 0 & 0 \\ i\mathcal{A}(r)\sin\vartheta\cos\varphi & 0 & -\chi r\sin\varphi & -r\chi\sin\vartheta\cos\vartheta\cos\varphi \\ i\mathcal{A}(r)\sin\vartheta\sin\varphi & 0 & \chi r\cos\varphi & -r\chi\sin\vartheta\cos\vartheta\sin\varphi \\ i\mathcal{A}(r)\cos\vartheta & 0 & 0 & \chi r\sin^2\vartheta \end{pmatrix}, \quad \chi = \pm 1,$$

$$\mathrm{d}s^2 = \mathcal{A}(r)^2\mathrm{d}t^2 - \mathcal{B}(r)^2\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2.$$

Though the tetrad is complex, both the torsion scalar and the boundary term are real

Exact solution:

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q}{r^2}\right)dt^2 - \left(\frac{2Mr - Q - r^2}{2Q - r^2}\right)^{-1} dr^2 - r^2 d\Omega^2$$

Q is a constant that it is not related to the electromagnetic charge, the spherically solution can have Q < 0 as well

Modified Newtonian Gravity:

- Modified Newtonian dynamics (MOND)

Adjust Newton’ s second law of motion (F = ma) by inserting a general function

$$F(a) = m\mu\left(\frac{a}{a_0}\right)a\,, \qquad \mu(x \gg 1) \approx 1\,,\ \mu(x \ll 1) \approx x\,.$$

This function predicts the observed flat rotation curves in the outskirts of galaxies, while still reproducing the Newtonian behaviour of the inner disc.

a ≫ a0 : Newtonian regime where F = ma

a ≪ a0 : Deep- MOND’ regime $F_{\mathrm{MOND}} = m\,a_{\mathrm{MOND}}^2/a_0$

- observed gravitational acceleration

$$g_{\mathrm{obs}}(g_{\mathrm{bar}}) = \frac{g_{\mathrm{bar}}}{1 - e^{-\sqrt{g_{\mathrm{bar}}/a_0}}} \qquad g_{\mathrm{obs}}(r) = \frac{G\,[4\Delta\Sigma_{\mathrm{obs}}(r)\,r^2]}{r^2} = 4G\Delta\Sigma_{\mathrm{obs}}(r)$$

- Aether Scalar Tensor (AeST) theory $\Phi = \tilde{\Phi} + \chi$

THANKS FOR
ATTENTION !